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# Lagrange Function of Charge in the Concept of the Scalar-Vector Potential

F. F. Mende

B. Verkin Institute for Low Temperature Physics and Engineering NAS Ukraine, Kharkov, Ukraine

**Email address**

mende\_fedor@mail.ru

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**Abstract**

The methods of the solution of the problems of mechanics is Lagrange formalism. By function of Lagrange or Lagrangian in the mechanics is understood the difference between the kinetic and potential energy of the system of in question if we integrate Lagrangian with respect to the time, then we will obtain the Gamilton first main function, called action. In the general case kinetic energy of system depends on speed, and potential energy depends on coordinates. With the condition of the conservatism of this system Lagrange formalism assumes least-action principle, when system during its motion selects the way, with which the action is minimum. However, the record of Lagrangian, accepted in the electrodynamics does not entirely satisfy the condition of the conservatism of system. The vector potential, in which moves the charge, create the strange moving charges, and the moving charge interacts not with the field of vector potential, but with the moving charges, influencing their motion. But this circumstance does not consider the existing model, since vector potential comes out as the independent substance, with which interacts the moving charge. Moreover, into the generalized momentum of the moving charge is introduced the scalar product of its speed and vector potential, in which the charge moves. But this term presents not kinetic, but potential energy, which contradicts the determination of pulse in the mechanics. With these circumstances are connected those errors, which occur in the works on electrodynamics. In the work it is shown that use of a concept of scalar- vector potential for enumerating the Lagrangian of the moving charge gives the possibility to exclude the errors, existing in the contemporary electrodynamics.

**1. Introduction**

The methods of the solution of the problems of mechanics is Lagrange formalism. By function of Lagrange or Lagrangian in the mechanics is understood the difference between the kinetic and potential energy of the system of in question

$$L = W_k(t) - W_p(t).$$

If we integrate Lagrangian with respect to the time, then we will obtain the Gamilton first main function called action. Since in the general case kinetic energy depends on speeds, and potential - from the coordinates, action can be recorded as

$$S = \int_{t_1}^{t_2} L(x_i, v_i) dt$$

With the condition of the conservatism of this system Lagrange formalism assumes least-action principle, when system during its motion selects the way, with which the

action is minimum.

In the electrodynamics Lagrangian of the charged particle, which is moved with the relativistic speed, is written as follows [1]:

$$L = -mc^2 \sqrt{1 - \frac{v^2}{c^2}} - q(\varphi + \mu_0(\vec{v}\vec{A}_H)) \quad (1.1)$$

This expression will be written down for the nonrelativistic speeds:

$$L = \frac{mv^2}{2} - q(\varphi + \mu_0(\vec{v}\vec{A}_H))$$

where  $q$ ,  $m$ ,  $\vec{v}$  are charge mass and the velocity of charge,  $c$  is the speed of light,  $\mu_0$  is magnetic permeability of vacuum,  $\varphi$  is scalar potential of electric field,  $\vec{A}_H$  is the vector potential of magnetic field, in which it moves with particle. This expression and further all relationships are written in the system of the units SI. However, the record of Lagrangian, accepted in the electrodynamics does not entirely satisfy the condition of the conservatism of system. The vector potential, in which moves the charge, create the strange moving charges, and the moving charge interacts not with the field of vector potential, but with the moving charges, influencing their motion. But this circumstance does not consider the existing model, since vector potential comes out as the independent substance, with which interacts the moving charge.

In the work [2] are located misunderstanding, on p. 279 read: "Therefore even in the relativistic approximation Lagrange's function in the electromagnetic field cannot be represented in the form differences in the kinetic and potential energy" (end of the quotation).

In the relationship (1.1), of the author it confuses the term, that contains the scalar product of the charge rate and vector potential, and he does not know, to what form of energy it to carry.

Among other things, this uncertainty is not over, and Landau works [3]. The introduction of the Lagrange function and moving charge in this work on paragraphs 16 and 17. With the introduction of these concepts in paragraph 16 is done the following observation: "The following below assertions it is necessary to examine to a considerable degree as the result of experimental data. The form of action for the particle in the electromagnetic field cannot be established on the basis only of general considerations, such as the requirement of relativistic invariance. (latter it would allow in action also the member of form integral of Ads, where A scalar function" (end of the quotation).

But with the further consideration of this question of any experimental data the author does not give and it is not completely understandable, on what bases Lagrangian's function introduces in the form (1.1). It is further - it is still worse. Without understanding the physical essence of Lagrangian, and in fact guess its (see relationship (17.4) into

[3]), the author immediately includes the potential part (the scalar product of speed and vector potential) in generalized momentum, and then for finding the force is differentiated on the coordinate of Lagrangian, calculating gradient from this value (see relationship after equality (18.1) of [3]). But, finding gradient from the work indicated, the author thus recognizes his potential status.

In the mechanics by pulse is understood the work of the mass of particle to its speed. Multiplying pulse on the speed, mechanical energy is derived. In the electrodynamics, in connection with the fact that the charge has a mass; also is introduced the concept of angular impulse. But this not all. Is introduced also the concept of the generalized momentum

$$\vec{P} = m\vec{v} + q\vec{A},$$

When to the angular impulse is added the work of charge to the vector potential of magnetic field, in which moves the charge. Moreover even with the insignificant magnetic fields this additive considerably exceeds angular impulse. If generalized momentum scalar was multiplied by the speed

$$\vec{v}\vec{P} = m(\vec{v})^2 + q\vec{v}\vec{A}, \quad (1.2)$$

That angular impulse will give kinetic energy. Scalar product of speed and vector potential will also give energy, here only this energy proves to be not kinetic, but potential. Here and is obtained, when it enters into the composition of energy of the moving charge and kinetic, and potential energy. With this is connected the incomprehension of physical nature of last term in the relationship (1.2), the having place in the work [2].

We already said that the record of Lagrangian (1.1) does not in the form satisfy the condition of the conservatism of system. This is connected with the fact that the vector potential, entering this relationship, it is connected with the motion of the strange charges, with which interacts the moving charge. A change in the charge rate, for which is located Lagrangian, will involve a change in the speed of these charges, and energy of the moving charge will be spent to this. In order to ensure the conservatism of system, it is necessary to know interaction energy of the moving charge with all strange charges, including with those, on which depends vector potential. This can be made a way of using the scalar- vector potential.

## 2. Concept of Scalar-Vector Potential

Gertz not only rewrote Maxwell's equations in the terms of partial derivatives. It made mistakes only in the fact that the electrical and magnetic fields were considered the invariants of speed. But already simple example of long lines is evidence of the inaccuracy of this approach. With the propagation of wave in the long line it is filled up with two forms of energy, which can be determined through the currents and the voltages or through the electrical and magnetic fields in the line. And only after wave will fill with electromagnetic energy all space

between the generator and the load on it will begin to be separated energy. I.e. the time, by which stays this process, generator expended its power to the filling with energy of the section of line between the generator and the load. But if we begin to move away load from incoming line, then a quantity of energy being isolated on it will decrease, since. the part of the energy, expended by source, will leave to the filling with energy of the additional length of line, connected with the motion of load. If load will approach a source, then it will obtain an additional quantity of energy due to the decrease of its length. But if effective resistance is the load of line, then an increase or the decrease of the power expendable in it can be connected only with a change in the stress on this resistance. Therefore we come to the conclusion that during the motion of the observer of those of relatively already existing in the line fields on must lead to their change.

Being located in assigned IS, us interest those fields, which are created in it by the fixed and moving charges, and also by the electromagnetic waves, which are generated by the fixed and moving sources of such waves. The fields, which are created in this IS by moving charges and moving sources of electromagnetic waves, we will call dynamic. Can serve as an example of dynamic field the magnetic field, which appears around the moving charges.

As already mentioned, in the classical electrodynamics be absent the rule of the conversion of electrical and magnetic fields on upon transfer of one inertial system to another. This deficiency removes SR, basis of which are the covariant Lorenz conversions. With the entire mathematical validity of this approach the physical essence of such conversions up to now remains unexplained.

In this division will made attempt find the precisely physically substantiated ways of obtaining the conversions fields on upon transfer of one IS to another, and to also explain what dynamic potentials and fields can generate the moving charges. The first step, demonstrated in the works [4], was made in this direction a way of the introduction of the symmetrical laws of magnetoelectric and electromagnetic induction. These laws are written as follows [4-11]

$$\oint \vec{E}' dl' = - \int \frac{\partial \vec{B}}{\partial t} d\vec{s} + \oint [\vec{v} \times \vec{B}] dl' \tag{2.1}$$

$$\oint \vec{H}' dl' = \int \frac{\partial \vec{D}}{\partial t} d\vec{s} - \oint [\vec{v} \times \vec{D}] dl'$$

or

$$rot \vec{E}' = - \frac{\partial \vec{B}}{\partial t} + rot [\vec{v} \times \vec{B}] \tag{2.2}$$

$$rot \vec{H}' = \frac{\partial \vec{D}}{\partial t} - rot [\vec{v} \times \vec{D}]$$

For the constants fields on these relationships they take the form:

$$\vec{E}' = [\vec{v} \times \vec{B}] \tag{2.3}$$

$$\vec{H}' = - [\vec{v} \times \vec{D}]$$

In relationships (2.1-2.3), which assume the validity of the Galileo conversions, prime and not prime values present fields and elements in moving and fixed IS respectively. It must be noted, that conversions (2.3) earlier could be obtained only from Lorenz conversions.

The relationships (2.1-2.3), which present the laws of induction, do not give information about how arose fields in initial fixed IS. They describe only laws governing the propagation and conversion fields on in the case of motion with respect to the already existing fields.

The relationship (2.3) attest to the fact that in the case of relative motion of frame of references, between the fields  $\vec{E}$  and  $\vec{H}$  there is a cross coupling, i.e., motion in the fields  $\vec{H}$  leads to the appearance fields on  $\vec{E}$  and vice versa. From these relationships escape the additional consequences, which were for the first time examined in the work [7].

The electric field  $E = \frac{g}{2\pi\epsilon r}$  outside the charged long rod with a linear density  $g$  decreases as  $\frac{1}{r}$ , where  $r$  is distance from the central axis of the rod to the observation point.

If we in parallel to the axis of rod in the field  $E$  begin to move with the speed  $\Delta v$  another IS, then in it will appear the additional magnetic field  $\Delta H = \epsilon E \Delta v$ . If we now with respect to already moving IS begin to move third frame of reference with the speed  $\Delta v$ , then already due to the motion in the field  $\Delta H$  will appear additive to the electric field  $\Delta E = \mu \epsilon E (\Delta v)^2$ . This process can be continued and further, as a result of which can be obtained the number, which gives the value of the electric field  $E'_v(r)$  in moving IS with reaching of the speed  $v = n\Delta v$ , when  $\Delta v \rightarrow 0$ , and  $n \rightarrow \infty$ . In the final analysis in moving IS the value of dynamic electric field will prove to be more than in the initial and to be determined by the relationship:

$$E'(r, v_{\perp}) = \frac{gch \frac{v_{\perp}}{c}}{2\pi\epsilon r} = Ech \frac{v_{\perp}}{c}$$

If speech goes about the electric field of the single charge  $e$ , then its electric field will be determined by the relationship:

$$E'(r, v_{\perp}) = \frac{ech \frac{v_{\perp}}{c}}{4\pi\epsilon r^2}$$

where  $v_{\perp}$  is normal component of charge rate to the vector, which connects the moving charge and observation point.

Expression for the scalar potential, created by the moving charge, for this case will be written down as follows:

$$\phi'(r, v_{\perp}) = \frac{ech \frac{v_{\perp}}{c}}{4\pi\epsilon r} = \phi(r)ch \frac{v_{\perp}}{c} \tag{2.4}$$

where  $\phi(r)$  is scalar potential of fixed charge. The potential

$\phi'(r, v_{\perp})$  can be named scalar-vector, since it depends not only on the absolute value of charge, but also on speed and direction of its motion with respect to the observation point. Maximum value this potential has in the direction normal to the motion of charge itself.

### 3. Lagrange Formalism in the Concept of Scalar-Vector Potential

The scalar potential  $\phi(r)$  at the point of the presence of charge is determined by all surrounding charges  $g_j$  and is determined by the relationship:

$$\phi(r) = \sum_j \frac{1}{4\pi\epsilon} \frac{g_j}{r_j}$$

The potential, determined by the relationship creates each moving charge at the observation point (2.5).

If some quantity of moving and fixed charges surrounds this point of space, then for finding the scalar potential in the given one to point it is necessary to produce the summing up of their potentials:

$$\phi'(r) = \sum_j \phi(r_j) ch \frac{v_{j\perp}}{c} = \sum_j \frac{1}{4\pi\epsilon} \frac{g_j}{r_j} ch \frac{v_{j\perp}}{c}$$

Taking into account this circumstance Lagrangian of the charge  $e$ , which is found in the environment of the fixed and moving strange charges can be written down as follows:

$$L = -e \sum_j \frac{1}{4\pi\epsilon} \frac{g_j}{r_j} ch \frac{v_{j\perp}}{c}$$

In that the case, if the charge  $e$  itself moves relatively by selected IS with the speed  $v$ , that its Lagrangian is, as earlier, determined by the relationship (1. 1) with the only difference that as the speeds  $v_{j\perp}$  relative charge rates with respect to the charge are taken  $e$  and is added the term, that determines kinetic energy of charge. Lagrangian for the low speeds in this case takes the form:

$$L = \frac{mv^2}{2} - e \sum_j \frac{1}{4\pi\epsilon} \frac{g_j}{r_j} ch \frac{v_{j\perp}}{c}$$

This approach is deprived already of the deficiency indicated, since it satisfies the complete conservatism of system, since in Lagrangian are taken into account all interactions charge with its surrounding charges.

### 4. Conclusion

The methods of the solution of the problems of mechanics is Lagrange formalism. By function of Lagrange or Lagrangian in the mechanics is understood the difference between the kinetic and potential energy of the system of in question if we integrate Lagrangian with respect to the time,

then we will obtain the Gamilton first main function, called action. In the general case kinetic energy of system depends on speed, and potential energy depends on coordinates. With the condition of the conservatism of this system Lagrange formalism assumes least-action principle, when system during its motion selects the way, with which the action is minimum. However, the record of Lagrangian, accepted in the electrodynamics does not entirely satisfy the condition of the conservatism of system. The vector potential, in which moves the charge, create the strange moving charges, and the moving charge interacts not with the field of vector potential, but with the moving charges, influencing their motion. But this circumstance does not consider the existing model, since vector potential comes out as the independent substance, with which interacts the moving charge. Moreover, into the generalized momentum of the moving charge is introduced the scalar product of its speed and vector potential, in which the charge moves. But this term presents not kinetic, but potential energy, which contradicts the determination of pulse in the mechanics. With these circumstances are connected those errors, which occur in the works on electrodynamics. In the work it is shown that use of a concept of scalar- vector potential for enumerating the Lagrangian of the moving charge gives the possibility to exclude the errors, existing in the contemporary electrodynamics.

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