Online ISSN : 2249-4626
Print ISSN : 0975-5896
DOI : 10.17406/GJSFR

Global Journal

of Science Frontier Research: A

Physics and Space Science

Theorys Epilogue (CUFT)

Phenomenon of Kinetic Energy

Highlights

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Entanglement and Statistical Properties

Discovering Thoughts, Inventing Future

VOLUME 18 ISSUE 9 VERSION 1.0

Global Journal of Science Frontier Research: A
Physics & Space Science

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Volume 18 Issue 9 (Ver. 1.0)

Open Association of Research Society

Incorporation No.: 0423089
License No.: 42125/022010/1186
Registration No.: 430374
Import-Export Code: 1109007027
Employer Identification Number (EIN):
USA Tax ID: 98-0673427

Global Journals Inc.

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Sponsors: Open Association of Research Society
Open Scientific Standards

Publisher's Headquarters office

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945th Concord Streets,
Framingham Massachusetts Pin: 01701,
United States of America
USA Toll Free: +001-888-839-7392
USA Toll Free Fax: +001-888-839-7392

Offset Typesetting

Global Journals Incorporated
2nd, Lansdowne, Lansdowne Rd., Croydon-Surrey,
Pin: CR9 2ER, United Kingdom

Packaging & Continental Dispatching

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Contents of the Issue

Global Journal of Science Frontier Research: a
Physics and Space Science
Volume 18 Issue 9 Version 1.0 Year 2018
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Electric Fields in the Concept of the Scalar-Vector Potential

By F. F. Mende

Abstract- All dynamic laws of electrodynamics, connected with the motion of charge, follow from the concept of scalar- vector potential. From this concept follow the emission laws. These laws are introduced with the aid of the being late scalar vector potential. This approach gives the possibility to understand physics of the process of emission and to obtain all necessary laws of this process.

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GJSFR-A Classification: *FOR Code: 010599*

Strictly as per the compliance and regulations of:

Electric Fields in the Concept of the Scalar-Vector Potential

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I. Introduction

In the concept of scalar- vector potential it is possible to isolate, at least, three cases of determining the tension of the electric field, which characterize actually the different versions of this concept [1-7].

a) Nonpotential electric field

If speech goes about the electric field of the single charge then its electric field will be determined by the relationship:

$$\mathbf{E}=\frac{e}{4\pi\varepsilon_0\mathbf{r}^2}\left(\mathbf{e}_v+\mathrm{ch}\frac{\sqrt{\mathbf{v}^2}}{c}\mathbf{e}_\perp\right),$$

where $\mathbf{e}_v=\frac{\mathbf{r}\cdot\mathbf{v}}{\mathbf{v}^2\sqrt{\mathbf{r}^2}}\mathbf{v}$ – the projection on the direction of the vector $\mathbf{v}$ of the unit vector $\mathbf{e}_r=\mathbf{r}/\sqrt{\mathbf{r}^2}$, of collinear to the vector $\mathbf{r}$; $\mathbf{e}_\perp=\mathbf{e}_r-\mathbf{e}_v=\frac{\mathbf{r}}{\sqrt{\mathbf{r}^2}}-\frac{\mathbf{r}\cdot\mathbf{v}}{\mathbf{v}^2\sqrt{\mathbf{r}^2}}\mathbf{v}$ – the projection of the unit vector $\mathbf{e}_r=\mathbf{r}/\sqrt{\mathbf{r}^2}$, of collinear to vector $\mathbf{r}$, on the direction of normal to the vector $\mathbf{v}$, of that lying at one plane s $\mathbf{r}$.

Let us fix certain moment of time. Let us select the system of rectangular Cartesian space coordinates $OXYZ$ so that at this moment the time the origin of coordinates would coincide with the moving point charge e, axis OX would be directed posigrade of charge $\mathbf{v}$, and vector $\mathbf{r}$ it would lie in the plane XOY. Then vector $\mathbf{r}$ can be assigned by two coordinates x, also, y along the axes OX and OY respectively. In this case longitudinal (along the axis OX) and transverse (along the axis OY) of the component of the vector of the tension of electric field at point $\mathbf{r}$ they will be equal respectively

$$E_x=\frac{e}{4\pi\varepsilon_0}\cdot\frac{x}{\sqrt{\left(x^2+y^2\right)^3}};\ E_y=\frac{e}{4\pi\varepsilon_0}\mathrm{ch}\frac{\sqrt{\mathbf{v}^2}}{c}\cdot\frac{y}{\sqrt{\left(x^2+y^2\right)^3}}.$$

Integration of these components for the appropriate coordinates gives:

$$\int E_x dx=-\frac{e}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{x^2+y^2}}+C_1(y);\ \int E_y dy=-\frac{e}{4\pi\varepsilon_0}\mathrm{ch}\frac{\sqrt{\mathbf{v}^2}}{c}\cdot\frac{1}{\sqrt{x^2+y^2}}+C_2(x),$$

where $C_1(y)$ and $C_2(x)$ – integration constant.

With $\sqrt{\mathbf{v}^2}=0$ the electric field proves to be potential with the known potential

$$\varphi(x,y)=\frac{e}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{x^2+y^2}},$$

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but with $\sqrt{\mathbf{v}^2} \neq 0$ the potentiality of field it is disrupted, i.e., there does not exist this function of potential, through which it would be possible to express the field strength according to the formula

$$\mathbf{E} = -\nabla\varphi .$$

Therefore such electric field of the moving charge must be examined as the superposition

$$\mathbf{E} = \mathbf{E}_v + \mathbf{E}_\perp$$

longitudinal electric field $\mathbf{E}_v$ with the potential (let us name its longitudinal potential)

$$\varphi_v(x,y) = \frac{e}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{x^2+y^2}}$$

and transverse electric field $\mathbf{E}_\perp$ with the potential (let us name its transverse potential)

$$\varphi_\perp(x,y) = \frac{e}{4\pi\varepsilon_0}\operatorname{ch}\frac{\sqrt{\mathbf{v}^2}}{c}\cdot\frac{1}{\sqrt{x^2+y^2}},$$

the fields $\mathbf{E}_v$ $\mathbf{E}_\perp$ having nontrivial components only on OX and OY :

$$\mathbf{E}_v = \frac{\partial\varphi_v}{\partial x};\ \mathbf{E}_\perp = \frac{\partial\varphi_\perp}{\partial y}.$$

Therefore in this case it is more correct to speak not about the concept of the scalar potential of charge, which depends on the speed, but about the concept of invariant longitudinal scalar potential and transverse scalar potential depending on the speed.

b) Potential electric field of Mende

Let us fix certain moment of time. Let us select the system of rectangular Cartesian space coordinates $OXYZ$ so that at this moment the time the origin of coordinates would coincide with the moving point charge e , axis OY would be directed posigrade of charge $\mathbf{v}$, and vector $\mathbf{r}$, directed from one point of the concentration of charge to the next, at which the field is determined, it lay in the plane XOY . Then vector $\mathbf{r}$ can be assigned by two coordinates x , also, y along the axes OX and OY respectively.

Let us assume that electric field is the potential field, given by the scalar potential

$$\varphi(\mathbf{r}) = e\operatorname{ch}\left(v_\perp(\mathbf{r})/c\right)\Big/\left(4\pi\varepsilon_0 r\right),$$

where $\mathbf{r} = (x,y)$ – the radius-vector of the point, at which is determined the field; $r = \sqrt{\mathbf{r}^2} = \sqrt{x^2+y^2}$ – the distance between the charge and the point, at which is determined the field; $v_\perp(\mathbf{r})$ – the component of charge rate e , is normal k $\mathbf{r}$.

Let there be $x > 0$, $y > 0$, then:

$$v_\perp(x,y) = \frac{vx}{\sqrt{x^2+y^2}} = \frac{vx}{r};\ \varphi(x,y) = \frac{e}{4\pi\varepsilon_0\sqrt{x^2+y^2}}\operatorname{ch}\frac{vx}{c\sqrt{x^2+y^2}} = \frac{e}{4\pi\varepsilon_0 r}\operatorname{ch}\frac{vx}{cr}.$$

The strength of this field has transverse component $E_x(x,y)$ and longitudinal component $E_y(x,y)$, which they are determined from the formulas:

$$E_x(x,y) = -\frac{\partial\varphi(x,y)}{\partial x} = \frac{e}{4\pi\varepsilon_0 r^3}\left(x\operatorname{ch}\frac{vx}{cr} - \frac{vy^2}{cr}\operatorname{sh}\frac{vx}{cr}\right);$$

$$E_y(x,y) = -\frac{\partial\varphi(x,y)}{\partial y} = \frac{ey}{4\pi\varepsilon_0 r^3}\left(\operatorname{ch}\frac{vx}{cr} - \frac{vx}{cr}\operatorname{sh}\frac{vx}{cr}\right).$$

c) Potential electric field with the transversely deformed scalar potential

Let us select the system of rectangular Cartesian space coordinates in the manner that in the case 2. The electrostatic potential of charge takes the form:

$$\varphi(x,y) = e\Big/\left(4\pi\varepsilon_0 r\right) = e\Big/\left(4\pi\varepsilon_0\sqrt{x^2+y^2}\right),$$

where $\mathbf{r} = (x,y)$ – the radius-vector of the point, at which is determined the field; $r = \sqrt{\mathbf{r}^2} = \sqrt{x^2+y^2}$ – the distance between the charge and this point.

Let us assume that the dynamic potential of charge is the transversely deformed electrostatic potential. Then we obtain the potential electric field, described by the scalar potential,

$$\varphi_{\perp}(v,x,y)=\varphi\left(x/\mathrm{ch}(v/c),y\right),$$

where $v=\sqrt{\mathbf{v}^2}$ – the module of charge rate, moreover $\varphi_{\perp}(0,x,y)=\varphi(x,y)$.

Taking into account expression for the electrostatic potential, we obtain formula for the dynamic potential:

$$\varphi_{\perp}(v,x,y)=\frac{e\,\mathrm{ch}(v/c)}{4\pi\varepsilon_0\sqrt{x^2+y^2\,\mathrm{ch}^2(v/c)}}.$$

The transverse $E_{\perp x}(v,x,y)$ and longitudinal $E_{\perp y}(v,x,y)$ components of the field strength will be written down in the form

$$E_{\perp x}(v,x,y)=-\frac{\partial\varphi_{\perp}(v,x,y)}{\partial x}=\frac{ex\,\mathrm{ch}(v/c)}{4\pi\varepsilon_0\sqrt{\left(x^2+y^2\,\mathrm{ch}^2(v/c)\right)^3}};$$

$$E_{\perp y}(v,x,y)=-\frac{\partial\varphi_{\perp}(v,x,y)}{\partial y}=\frac{ey\,\mathrm{ch}^3(v/c)}{4\pi\varepsilon_0\sqrt{\left(x^2+y^2\,\mathrm{ch}^2(v/c)\right)^3}}.$$

With $x\neq 0$, $y=0$ we have:

$$E_{\perp x}(v,x,y)=\frac{e\,\mathrm{ch}(v/c)}{4\pi\varepsilon_0 x^2};\ E_{\perp y}(v,x,y)=0.$$

With $x=0$, $y\neq 0$ we have:

$$E_{\perp x}(v,x,y)=0;\ E_{\perp y}(v,x,y)=\frac{e}{4\pi\varepsilon_0 y^2}.$$

d) Conclusions of three cases examined are the following

Thus, three versions of the determination of electrical pour on in the concept of the scalar- vector potential proposed:

1. the nonpotential field, whose longitudinal component of tension does not depend on speed, but transverse – depends;
2. potential field in Mende's version;
3. potential field with the transversely deformed scalar potential.

Last two versions characterize the potential field, in which in general case both components depend on speed; however, for this field it is possible to isolate two extreme special cases for the points, at which this field is determined: if to point (at the coordinate system proposed these points lie on the axis the transverse component OY) of field it is absent into some, then longitudinal does not depend on speed; if to point (at the coordinate system proposed these points lie on the axis the longitudinal component OX) of field it is absent into some, then transverse depends on speed by the simple dependence, which is obtained by the multiplication of the corresponding strength of electrostatic field to the coefficient $\mathrm{ch}(v/c)$.

The potentiality of electric field means that during the motion in this field of trial charge along the locked trajectory the work is not accomplished. But if scalar potential changes in the time, then in the time changes electric field the, as a result of which this property of the absence of the perfect work, strictly speaking, it is not carried out. However, in this case for each assigned moment of time potential electric field (last two versions) can be defined as the undertaken with the opposite sign gradient of scalar potential.

An important difference in the first and third versions in the first version lies in the fact that occurs the lateral deformation of the strength of the field (it it leads to the nonpotentiality of field), and in the third – the lateral deformation the scalar potential field (it it leads to the dependence on the speed in general case of both components of the field strength).

In the present monograph the basis is undertaken the second version of concept, although the third version from a physical point of view is presented to us not less, but, possibly, by more promising, and the first version is also is sufficiently promising. In

connection with this, one of the most important tasks of further studies – the determination of version, most adequate of reality.

II. Laws Of The Electro-Electrical Induction

We will consider that the charge e accomplishes fluctuating motion along the axis y, in the environment of the origin of coordinates 0 (Fig. 13), moreover the amplitude of the fluctuations of charge is considerably lower than the distance from the charge to the observed point. According to the second version of concept, the scalar potential of charge at point with the abscissa x and the ordinate $y = 0$ (fixed point (1) in Fig. 1) depends on the component $v_\perp \approx v_y$ of its velocity, normal to the vector $\mathbf{r}$, connecting observation point and the moving charge:

$$\varphi = \frac{e\,\mathrm{ch}\left(v_\perp / c\right)}{4\pi\varepsilon_0 r} = \frac{e\,\mathrm{ch}\left(v_y / c\right)}{4\pi\varepsilon_0 r} \qquad (2.1)$$

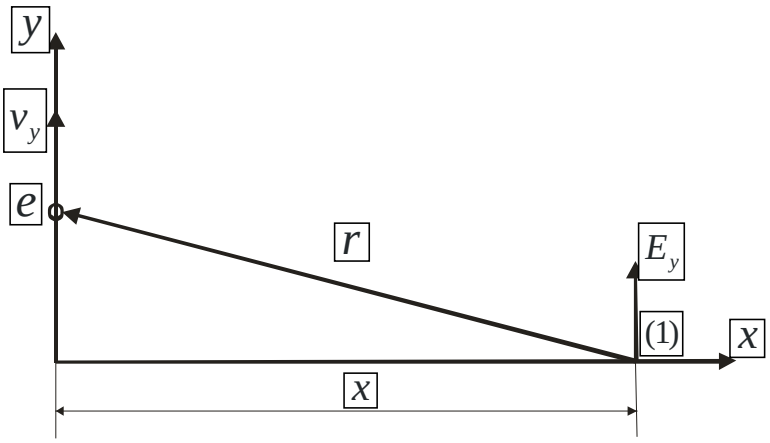

Fig. 1: Diagram of shaping of the induced electric field

According to relationship $\mathbf{E} = -\mathrm{grad}\,\varphi$, the projection on the axis y of the tension of the induced electric field at the same point is equal

$$E_y = -\frac{\partial\varphi}{\partial y} = -\frac{\partial}{\partial y}\frac{e}{4\pi\varepsilon_0 r(y)}\mathrm{ch}\frac{v_y}{c}.$$

Counting in view of the smallness of the amplitude of fluctuations radius-vector by constant, let us rewrite last equality in the form:

$$E_y = -\frac{e}{4\pi\varepsilon_0 cx}\frac{\partial v_y}{\partial y}\mathrm{sh}\frac{v_y}{c}. \qquad (2.2)$$

a) Taking into account that

$$\frac{\partial v_y}{\partial y} = \frac{\partial v_y}{\partial t}\frac{\partial t}{\partial y} = \frac{\partial v_y}{\partial t}\frac{1}{v_y},$$

we obtain from (2.2):

$$E_y = \frac{e}{4\pi\varepsilon_0 cx}\frac{1}{v_y}\frac{\partial v_y}{\partial t}\mathrm{sh}\frac{v_y}{c}. \qquad (2.3)$$

Counting the charge rate of much smaller speed of light and taking only first term of expansion $\mathrm{sh}\frac{v_y}{c} \cong \frac{v_y}{c}$, from (2.3) we have:

$$E_y = -\frac{e}{4\pi\varepsilon_0 c^2 x}\frac{\partial v_y}{\partial t} = -\frac{ea_y}{4\pi\varepsilon_0 c^2 x} \qquad (2.4)$$

where $a_y = \partial v_y / \partial t$ - acceleration of charge.

If we as the direction of emission take the vector, which lies at the plane xy, and which constitutes with the axis y the angle α, then relationship (2.4) takes the form:

$$E_y = -\frac{ea_y \sin\alpha}{4\pi\varepsilon_0 c^2 x}.$$

This relationship determines the radiation pattern of shaping of the induced electric field.

Let us introduce delay in the scalar- vector potential, considering as its conditioned propagation of field in this medium with the speed of light:

$$\varphi = \frac{e\,\mathrm{ch}\dfrac{v_y\left(t - r/c\right)}{c}}{4\pi\varepsilon_0 r} \qquad (2.5)$$

where $v_y\left(t - r/c\right)$ - component of the charge rate e, normal to the vector $\mathbf{r}$ at the moment of the time $t' = t - r/c$, r - distance between the charge and the point, at which t.

Again using a relationship $\mathbf{E} = -\mathrm{grad}\,\varphi$, let us find field at the same point:

$$E_y = -\frac{\partial\varphi}{\partial y} = -\frac{\partial}{\partial y}\frac{e}{4\pi\varepsilon_0 r(y)}\mathrm{ch}\frac{v_y\left(t - r(y)/c\right)}{c}.$$

Considering again radius-vector constant, we will obtain:

$$E_y = -\frac{e}{4\pi\varepsilon_0 cx}\frac{\partial v_y\left(t - x/c\right)}{\partial y}\mathrm{sh}\frac{v_y\left(t - x/c\right)}{c} \qquad (2.6)$$

b) Taking into account that

From (2.6) we obtain the complete emission law of the moving charge:

$$E_y(x,t) = \frac{e}{4\pi\varepsilon_0 cx}\frac{1}{v_y\left(t - x/c\right)}\frac{\partial v_y\left(t - x/c\right)}{\partial t}\mathrm{sh}\frac{v_y\left(t - x/c\right)}{c} \qquad (2.7)$$

Counting the charge rate of much smaller speed of light and taking only first term of expansion $\mathrm{sh}\dfrac{v_y\left(t - x/c\right)}{c} \cong \dfrac{v_y\left(t - x/c\right)}{c}$, from (2.7) we will obtain the wave equation, which defines both the amplitude, and phase response of the wave of electric field, radiated by the moving charge:

$$E_y(x,t) = -\frac{e}{4\pi\varepsilon_0 c^2 x}\frac{\partial v_y\left(t - x/c\right)}{\partial t} = -\frac{ea_y\left(t - x/c\right)}{4\pi\varepsilon_0 c^2 x}, \qquad (2.8)$$

where $a_y\left(t - x/c\right)$ - being late acceleration of charge.

After selecting by the direction of emission vector in the plane xy, component with the axis y angle α, from (2.8) we will obtain the relationship, which determines known radiation pattern of dipole source (complete diagram it is symmetrical relative to the axis y):

$$E_y(x,t,\alpha) = -\frac{ea_y\left(t - x/c\right)\sin\alpha}{4\pi\varepsilon_0 c^2 x} = -\frac{1}{\varepsilon_0 c^2}\frac{\partial A_H\left(t - x/c\right)}{\partial t} = -\mu_0\frac{\partial A_H\left(t - x/c\right)}{\partial t} \qquad (2.9)$$

Known in the electrodynamics being late vector potential

$$\frac{ev_z\left(t - x/c\right)}{4\pi x} = A_H\left(t - x/c\right)$$

it is here introduced not heuristic and phenomenologically, but on the basis of the being late scalar- vector potential. In Maxwell equations the electric fields of wave are vortex, while in this concept – are gradient.

Still one possibility of relationship (2.9) – the description of the dipole emission of electromagnetic waves by the charges, which are varied in the electric field. Time derivative of the dipole moment (vector $\mathbf{d}$ it is directed from the negative charge toward the positive)

$$\mathbf{p} = e\mathbf{d} \qquad (2.10)$$

it is connected with the current:

$$e\mathbf{v} = e\frac{\partial\mathbf{d}}{\partial t} = \frac{\partial\mathbf{p}}{\partial t},\ \mathbf{v} = \frac{1}{e}\frac{\partial\mathbf{p}}{\partial t},\ \mathbf{a} = \frac{\partial\mathbf{v}}{\partial t} = \frac{1}{e}\frac{\partial^2\mathbf{p}}{\partial t^2}.$$

The substitution of last equality for $\mathbf{a}$ in (2.9) gives the known [8] emission law of the being varied dipole

$$\mathbf{E} = -\frac{1}{4\pi r\varepsilon_0 c^2}\frac{\partial^2\mathbf{p}(t - r/c)}{\partial t^2}. \qquad (2.11)$$

Thus, the field of the being varied electric dipole is determined by the superposition of electrical induction pour on the emissions (2.8), (2.9), (2.11), of those connected with the acceleration of charge, and electrical pour on static dipole, that are changed according to dependence on the time of the distance between the charges. The process of emission is connected with the transformation of energy pour on static dipole into the energy pour on emissions.

Laws (2.8), (2.9), (2.11) do not use magnetic fields and vector potentials, showing thus the fundamentality of electrical and the second-rateness of magnetic pour on.

Relationship (2.9) makes it possible to obtain the laws of reflection and scattering of electrical pour on by the totality of the charges, set to the forced motion by external (strange) electric field. The superposition of electrical pour on, radiated by all charges, it is electrical wave.

The acceleration of charge under the action of strange electric field $E'_y = E'_{y0}\sin\omega t$ is equal

$$a = -\frac{e}{m}E'_{y0}\sin\omega t.$$

Taking into account this relationship (18.9) assumes the form

$$E_y(x,t,\alpha) = \frac{e^2\sin\alpha}{4\pi\varepsilon_0 c^2 mx}E'_{y0}\sin\omega(t-\frac{x}{c}) = \frac{K}{x}E'_{y0}\sin\omega(t-\frac{x}{c}) \quad (2.12)$$

where the determining ability of charge to re-emit external field the value

$$K = \frac{e^2\sin\alpha}{4\pi\varepsilon_0 c^2 m}.$$

Let us name the coefficient of scattering (re-emission) charge in the assigned direction.

The current wave of the displacement accompanies the wave of electric field:

$$j_y(x,t) = \varepsilon_0\frac{\partial E_y}{\partial t} = -\frac{e\sin\alpha}{4\pi c^2 x}\frac{\partial^2 v_y(t-x/c)}{\partial t^2}.$$

If charge accomplishes its motion under the action of the electric field $E' = E'_0\sin\omega t$, then bias current in the distant zone will be written down as

$$j_y(x,t) = -\frac{e^2\omega}{4\pi c^2 mx}E'_{y0}\cos\omega(t-x/c). \quad (2.13)$$

The wave of electric field (18.12) and laggard behind it on $\pi/2$ the current wave of displacement (2.13) form the wave that named there electriccurent.

In parallel with the electrical waves it is possible to introduce magnetic waves, if we assume that

$$\mathbf{j} = \varepsilon_0\frac{\partial \mathbf{E}}{\partial t} = \mathrm{rot}\,\mathbf{H} \quad (2.14)$$

$$\mathrm{div}\,\mathbf{H} = 0.$$

Such magnetic field – is vortex. Comparing (2.13) and (2.14) we obtain:

$$\frac{\partial H_z(x,t)}{\partial x} = \frac{e^2\omega\sin\alpha}{4\pi c^2 mx}E'_{y0}\cos\omega\left(t-\frac{x}{c}\right).$$

Integrating this relationship on the coordinate, we find the value of the magnetic field

$$H_z(x,t) = \frac{e^2\sin\alpha}{4\pi cmx}E'_{y0}\sin\omega\left(t-\frac{x}{c}\right). \quad (2.15)$$

Thus, relationship (2.12), (2.13) and (2.15) can be named the laws of electrical induction, since. they give the direct coupling between the electric fields, applied to the charge, and by fields and by currents induced by this charge in its environment. Charge itself comes out [v] in the role of the transformer, which ensures this reradiation. Magnetic field (2.15) is directed normally both toward the electric field and toward the direction of propagation. In this case it is executed:

$$\frac{E_y(x,t)}{H_z(x,t)} = \frac{1}{\varepsilon_0 c} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = Z,$$

where Z - wave drag of free space.

Wave drag determines the active power of losses on the single area, located normal to the direction of propagation of the wave:

$$P = ZE_{y0}^2/2.$$

This relationship determines the power flux of the electriccurent wave through this area according to Poynting theorem.

Thus, any wave process in the fields of electromagnetic nature is reduced to the electriccurent waves in the space, characterized by its wave drag. This examination does not require the attraction of the concepts of magnetic field and its vector potential. Nevertheless, these concepts can be additionally introduced, clearly dividing fields to the gradient (electrical) and the vortex (magnetic) according to Helmholtz's theorem, which says, that any single-valued and continuous vectorial field $\mathbf{F}$, turning into zero at infinity, can be represented, and besides uniquely, in the form the sum of the gradient of a certain scalar function φ and rotor of a certain vector function $\mathbf{C}$, divergence of which is equal to zero:

$$\mathbf{F} = \text{grad}\,\varphi + \text{rot}\,\mathbf{C},\ div\mathbf{C} = 0.$$

thus, the construction of electrodynamics should have been begun from the acknowledgement of the dependence of scalar potential on the speed. But nature very deeply hides its secrets, and in order to come to this simple conclusion, it was necessary to pass way by length almost into two centuries.

The grit, which so harmoniously were erected around the magnet poles, in a straight manner indicated the presence of some power pour on potential nature, but to this they did not turn attention; therefore it turned out that all examined only tip of the iceberg, whose substantial part remained invisible of almost two hundred years.

Is exponential quotation from [9]: «but in what does consist the basic initial reason for the discrepancy of the built by Maxwell electrodynamics? For the single-valued answer to this question… it should be noted that even in its time of amperes, Grossman, Gauss, Lentz, Neumann, Veber, Riemann and other they stood on the point of view, that, without being turned to the concept "of magnetic field", any magnetic interactions can be reduced to usual interactions of current elements or moving charges… in the electrodynamics repossessed then the point of view of Faraday and Maxwell, that the electrical and "magnetic" fields are the independent physical essences, although connected together. In the prevailing then historical situation given, erroneous from a physical point of view, assumptions predetermined by themselves entire further motion of the development of electrodynamics with the deliberately placed into it insoluble contradictions and the paradoxes». And further there: «for the noncontradictory reflection of the physical essence of the laws of electromagnetism necessary to completely forego any concepts "magnetic field" as certain independent physical essence… for determining the forces of interaction of moving in the physical vacuum of real space electric charges completely sufficient to consider the deformation of electrical pour on these charges, caused by the trivial effects of the being late potentials… To there remains only be surprised at the sagacity of the ampere, which warned that if we in the electrodynamics do not forego ourselves the concept "magnet", then subsequently this threatens by incredible confusion in the theory».

One should assume that the basis of the overwhelming majority of the static and dynamic phenomena of electrodynamics – the assuming dependence of the scalar potential of charge on the speed of its motion formula

$$\mathbf{E}' = -\mu \frac{d\mathbf{A}_H}{dt}$$

or the analogous to it formula, which reflects the different version of the concept of the scalar- vector potential. From this formula it follows and static interaction of charges, and laws of power interaction in the case of their mutual motion, and emission laws and scattering. This approach made it possible to explain from the positions of classical electrodynamics such phenomena as phase aberration and the transverse Doppler effect, which within the framework the classical electrodynamics of explanation did not find. After entire aforesaid it is possible to remove construction forests, such as magnetic field and magnetic vector potential, which do not allow here already almost two hundred years to see the building of electrodynamics in entire its sublimity and beauty.

Let us point out that one of the fundamental equations of induction

$$rot\mathbf{A}_H = \mathbf{H}$$

could be obtained directly from the Ampere law, still long before appeared Maxwell equations. The Ampere law, expressed in the vector form, determines magnetic field at the point

$$\mathbf{H} = \frac{1}{4\pi}\int \frac{Id\mathbf{l}\times\mathbf{r}}{r^3},$$

where I - current in the element $d\mathbf{l}$, $\mathbf{r}$ - vector, directed from $d\mathbf{l}$ to the point.

It is possible to show that

$$\frac{d\mathbf{l}\times\mathbf{r}}{r^3} = \mathbf{grad}\left(\frac{1}{r}\right)\times d\mathbf{l} = \mathbf{rot}\left(\frac{d\mathbf{l}}{r}\right) - \frac{1}{r}\mathbf{rot}d\mathbf{l} = \mathbf{rot}\left(\frac{d\mathbf{l}}{r}\right)$$

therefore finally we will obtain:

$$\mathbf{H} = \text{rot}\int I\left(\frac{d\mathbf{l}}{4\pi r}\right) = \text{rot}\,\mathbf{A}_\mathbf{H},$$

where

$$\mathbf{A}_\mathbf{H} = \int I\left(\frac{d\mathbf{l}}{4\pi r}\right). \qquad (2.16)$$

The remarkable property of this expression is that that the vector potential depends from the distance to the observation point as $1/r$. Specifically, this property makes it possible to obtain emission laws.

Since. $I = gv$, where g - a charge of the unit of the length of conductor, from (2.16) we have:

$$\mathbf{A_H} = \int \frac{gv\ d\mathbf{l}}{4\pi r}.$$

For the single charge e this relationship takes the form:

$$\mathbf{A_H} = \frac{e\mathbf{v}}{4\pi r}.$$

Since

$$\mathbf{E} = -\mu \frac{\partial \mathbf{A_H}}{\partial t},$$

that in the general case we have (here a - the acceleration of charge):

$$\mathbf{E} = -\mu \int \frac{g\dfrac{\partial v}{\partial t}\ d\mathbf{l}}{4\pi r} = -\mu \int \frac{ga\ d\mathbf{l}}{4\pi r} \qquad (2.17)$$

and we will obtain for a special case of single charge:

$$\mathbf{E} = -\frac{\mu e\mathbf{a}}{4\pi r}. \qquad (2.18)$$

If we in (2.17), (2.18) consider the propagation time delay $\left(t - r/c\right)$ of potentials and that the fact that for the vacuum $\mu = 1/\left(\varepsilon_0 c^2\right)$, these equalities will take the form:

$$\mathbf{E} = -\mu \int \frac{ga(t - r/c)\ d\mathbf{l}}{4\pi r} = -\int \frac{ga(t - r/c)\ d\mathbf{l}}{4\pi\varepsilon_0 c^2 r}, \qquad (2.19)$$

$$\mathbf{E} = -\frac{e\mathbf{a}(t - r/c)}{4\pi\varepsilon_0 c^2 r}, \qquad (2.20)$$

Relationships (2.19) (2.20) represent wave equations. Let us note that these equations - this solution of Maxwell' equations, but in this case they are obtained directly from the Ampere law, not at all coming running to Maxwell equations. To there remains only present the question, why electrodynamics in its time is not banal by this method?

Given examples show, as electrodynamics in the time of its existence little moved. The phenomenon of electromagnetic induction Faraday opened into 1831 and already almost 200 years its study underwent practically no changes, and the physical causes for the most elementary electrodynamic phenomena, until now, were misunderstood. Certainly, for his time Faraday was genius, but that they did make physics after it? Even such geniuses, as Maxwell and Hertz, did not establish fundamental role in the entire classical electrodynamics of the dependence of the scalar potential of charge on his relative speed. Subsequently such scientists as Nikolayev and Marinov their theoretical and experimental studies were conducted in this direction, but proper acknowledgement as scientific association these works so did not obtain.

III. Conclusion

All dynamic laws of electrodynamics, connected with the motion of charge, follow from the concept of scalar- vector potential. In the article it is shown that alsoz of this concept follow the emission laws. These laws are introduced with the aid of the being late scalar vector potential. This approach gives the possibility to understand physics of the process of emission and to obtain all necessary laws of this process.

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Global Journal of Science Frontier Research: a
Physics and Space Science
Volume 18 Issue 9 Version 1.0 Year 2018
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Effect of Parametric Variations on Electromigration in Integrated Circuits

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Abstract- This research paper focuses on the effects of electromigration in integrated circuits at the nanoscale domain. This is an investigative work that shows how various process and parametric variation effects on electromigration. With integrated circuits reaching in the nanoscale domain, electromigration is becoming more of a prominent problem. Being able to find changes into the integrated circuits to provide a better electromigration performance is crucial for future emerging nanotechnologies. Therefore, this paper will go through previous research work to show the evolution of Black's equation and see if Black's equation could use on nanoscale integrated circuits. Also, it will be showing future iterations of the equation and comparing them with constant variables. Besides, a comparison of aluminum and copper interconnects and how electromigration happens differently are also discussing in this paper. Then a conclusion on how Black's equation could use with nanoscale technologies to predict the time during the occurrence of electromigration.

Keywords: *activation energy, current density, electromigration, interconnects, integrated circuits, mean time failure.*

GJSFR-A Classification: *FOR Code: 090604*

Strictly as per the compliance and regulations of:

Effect of Parametric Variations on Electromigration in Integrated Circuits

Drew K. Tallman [α] & Muhammad S. Ullah [σ]

Abstract- This research paper focuses on the effects of electromigration in integrated circuits at the nanoscale domain. This is an investigative work that shows how various process and parametric variation effects on electromigration. With integrated circuits reaching in the nanoscale domain, electromigration is becoming more of a prominent problem. Being able to find changes into the integrated circuits to provide a better electromigration performance is crucial for future emerging nanotechnologies. Therefore, this paper will go through previous research work to show the evolution of Black's equation and see if Black's equation could use on nanoscale integrated circuits. Also, it will be showing future iterations of the equation and comparing them with constant variables. Besides, a comparison of aluminum and copper interconnects and how electromigration happens differently are also discussing in this paper. Then a conclusion on how Black's equation could use with nanoscale technologies to predict the time during the occurrence of electromigration.

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I. Introduction

Electromigration is a phenomenon that has been around for almost a century. This concept will happen on an interconnect line segment that is under Constant use. These interconnect lines are the ones that transport supply voltage (Vcc) to the chip. Electrons will become material carriers and transport material to one end of the line causing hillocks/ fringes, with nucleation voids at another end. With the gigantic scaling down of VLSI circuits, the anxiety of electromigration will grow due to an increase in current density. This electromigration is caused by the internal gap or defect in the wire. The contact or via may break or become a short circuit to another layer of wire. Power and ground networks are very prone to electromigration because of unidirectional current compare to regular signal wires that carry bidirectional currents. Besides, average current density, the rate of electromigration depends on the temperature, the crystal structure, grain transport properties of the conducting materials.

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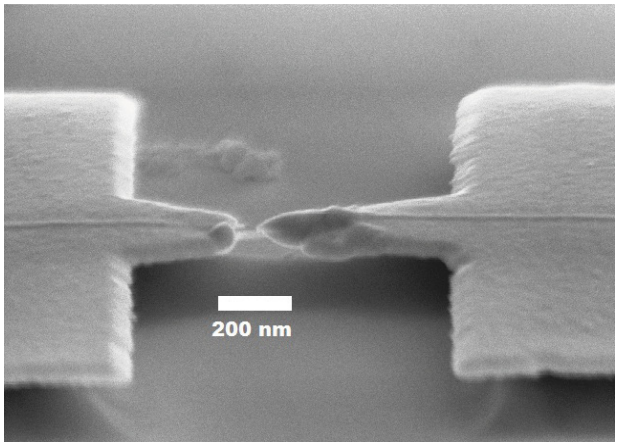

Fig. 1: The conception of Electromigration

The circuit designer can only effectively control current density. Therefore, keeping the current density within limit is essential to prevent electromigration. The concept of electromigration shows in Figure 1. This paper will show a history of the findings of electromigration. This is an investigative work to find what factors must need to change in order to have a better electromigration performance and reliability of Black's equation with the nanoscale technologies.

II. Previous Research Work On Electromigration

Starting with 1967 when interconnect lines were made of aluminum. The first person who developed a mathematical model for predicting electromigration was J.R. Black while he worked for Motorola. Black studied the effects of electromigration on different aluminum strips. Black tested these strips using accelerated temperature testing. This method of testing was putting the aluminum strips under a constant temperature and a range of current densities [1, 2]. Black's equation is the current density squared along with the equation for mass transport. Black's initial model is as follows on equation (1)

$$\frac{1}{MTF} = AJ^2 \exp\left(-\frac{Q}{kT}\right) \qquad (1)$$

where MTF is the median time to failure, *A* is a constant that reflects the cross-sectional area of the film along

with many other factors such as, the volume resistivity of the metal, the electro-free time between collisions or the electron free path and average velocity, ionic scattering cross section for electrons, the self-diffusion frequency factor of aluminum in aluminum materials, and a factor relating rate of mass transport of MTF, *J* is the current density, *Q* is the activation energy, *k* is Boltzmann's constant and *T* is temperature. He confirmed that the relationship of current density by performing experiments with current density ranges are larger than 5:1. It is determined that the activation energy was dependent upon the overall structure of the film. For example, his experiments were carried out with 3 different crystallite films. For small crystallite films (1.2μ), the activation energy was .48, and for large crystallite films (8 μ) it was 0.84. Black's experiments done at a constant temperature of 235°C, and a current density of 1.3 X 10^5 A/cm for a period of 230 hours. This experiment taken more time than normal time for the process of electromigration to occur. Black found voids at the positive ends of resistor contact points along with hillocks and fringes that happening at the negative regions which shows a transport of material. It concluded with the experimental data which was showing voids and hillocks in addition to providing charts of functions between current and temperature [1, 2, 3].

In 1971, Blair made changes to Black's equation [4]. Blair made changes to Black's equation by having a scaled factor of nn for current density instead of having the current density scale by a factor of 2 which shown in equation (2).

$$MTF = AJ^{-n} \exp\left(\frac{Q}{kT}\right) \quad (2)$$

This change made because Blair and Hartman found that MTF depends on the divergence of atomic flux which determined by the product of the current density and a constant temperature for a set amount of time. The value of n was considered to be in between 1 and 2.

The value of *n* has been a abundant controversy. Was it 1, 1.5, or upwards to 20? Many papers indicated their findings of what n should be. Shatzkes and Lloyd showed that there is a discrepancy in how n is evaluated [12]. In 1986, as IC lines were getting smaller, aluminum was found to have flaws with maintaining a high resistivity and thermal capability to be under pressure for a long lifetime. Developers started to dope the aluminum connecting lines with copper. Copper had a higher resistivity and shown to prolong the lifetime of an interconnect line which revealed that it increases in electromigration performance. M. Shatzkes and J.R Lloyd have found that there were errors in (2).

More specifically, how the value of n found in the original model due not to consider Joule heating, which could scale the exponent to a higher value than 2. Also, Joule heating becomes a temperature gradient induced flux divergence which shown in their model in equation (3).

$$MTF = BT^2J^{-2} \exp\left(\frac{Q}{kT}\right) \quad (3)$$

The value B found with another model that shown in (4)

$$B = \left(\frac{2C_f}{D_0}\right)\left(\frac{k}{Z^*e\rho}\right)^2 \quad (4)$$

Where C_f is a critical value that represents the vacancy concentration, D_0 is a pre-exponential factor for grain-Boundary self-diffusivity, Z* is effective charge of magnitude and direction in momentum exchange between a conducting electron and diffusing metal atoms, ρ is the material's resistivity, e is the mass of an electron. Shatzkes and Lloyd then found the activation energy that calculated higher than the one anticipated with equation (1). The value of n determined to be closer to 2 after experimental data using in both equations. Shatzkes and Lloyd proposed that the activation energy with the added T^2 value produced MTF values that were as good or better with their data [12].

A paper was done by M.L Dreyer, K.Y. Fu, and C. J Varker, introduced a new contribution to the calculations on diffusion components for mass transport [7]. In particular, how line widths compared to the median grain size affected the MTF value. Providing models that considered the grain boundary and lattice components of diffusion. This paper's purpose is not to cover their mathematical formula, but more so highlight that if a line width is better than the median grain size will produce a longer lifetime. By providing experimental data, in fact, a line width depends on the electromigration activation energy, along with temperature change.

In 1997, aluminum interconnects were beginning to become hard to come by copper. Copper found to be a better material due to a higher resistivity and having a much better-conducting metal. Copper and aluminum are both prone to electromigration. But copper has a much better electromigration performance, so IC's with copper interconnect lines will have a longer lifetime. Copper is also able to be layered upon itself giving more functionality to an IC. Even though electromigration happens in both copper and aluminum, they have different properties which reflect the way of electromigration which will arise [13]. Aluminum that undergoes electromigration will have something called triple points. Triple points are points where three aluminum crystals meet together. When

voids happen, two (2) of these points will become material current carriers that leaving a hole behind, and one of those points will also be a material current carrier that will bring in material, but there is a discrepancy. On the other side of the line where hillocks occur, the opposite happens. Two current carriers are bringing material in, with one current carrier taking material away. In copper, electromigration will occur by boundaries of an interconnect line where found either via connect on right side or via above through in the transmission line.

In 2003, K. N. Tu did a study on electromigration in aluminum, copper, and solder joints and compared the different electromigration effects on their operating temperature of 100 °C [5]. He analyzed on MTF, separating into current density, activation energy, and how void nucleation/ hillocks effect on cathodes. Their experiments showed how "low-k copper interconnects" have a severe problem of electro-migration happening at small current densities due to current crowding. This is because of the current density gradient that acting like a normal current flow. For MTF, the time taken for a void to appear is more significant in aluminum interconnects because of the way it structured and copper had electromigration happening at surfaces [8].

In 2007, Cher Ming Tan, Yuejin Hou, and Wei Li revisited black's equation to model for narrow interconnects. They proposed a new driving force formula which considered three different stress with induced migrations. The first being electron-wind forced migration, temperature gradient induced migration and stressed induced electromigration. The most important part of this paper for our purpose is a change in the interconnecting line circuit. They were able to calculate a critical reservoir length that reduced the probability of a precarious electromigration void. The reservoir is an extension on the interconnect line that goes pass through the via. This is to allow a distribution of stress on the line [9].

In 2008, Armin H. Fischer and Alexander von Glasow showed the different electromigration failure in copper interconnect. The categorized them in an early failure and late failure. Early failure was indicating that the void has grown underneath the via, causing a disconnect. Late failure specifies that a void has developed within the transmission line or in their words "trench-voiding" which happens just above the via. They then took multiple samples with a pre-cleaning, which is a SiN layer added to copper. They found that the pre-cleaning showed that early mode has a small slit-like a void that appears, then the late failure mode shows a larger void. The early failure mode in comparison to MTF had an n value of 1.1, and the late failure mode had an n value of 1.5 as more current in a smaller area of the wire [10].

In 2017, the most recent study of all previous works focused on nanoscale devices and what factors needed to consider for electromigration modeling. They provided a model that would show how dominant the grain boundaries and material interfaces effected on electromigration. They used TiN with copper grains that are for via connections. The scattering effect is showing by providing a lot of models for current density traveling through interconnecting lines.

When grain boundaries considered, the lifetime of an IC changed by a factor of 5. This show that grain boundaries need to consider when dealing with nanoscale cross-sectional areas in interconnecting circuits. Different material interfaces need to be considered for the different scattering effects when current is applied. Current scatter and crowding will be the ultimate downfall of a nanoscale IC if grain boundaries and material interfaces did not put into consideration into the MTF equation [11].

III. Simulation Results and Analysis

The simulations are done using MATLAB software. The following simulations has show an array of how different variables effect on both in equation (1) and equation (2). In equation (1), the following equation (5) used which taken from Black's experiment.

$$MTF = \frac{w\ t\ exp\left(\frac{0.84}{kT}\right)}{5 * 10^{-13}\ J^2} \tag{5}$$

Where ww indicating the width of the line, and tt indicating the thickness. These were scaled by having the area. The multiplication of both width and thickness show the behavior of MTF with a change in an area which is seen from Figure 2. Information from each line in figure 2 represents a change in temperature. With the top utmost line being at 350°C, each line is showing a change in 3 degrees up to 360°C. This alone proves along with equation (5) that the electromigration depends on temperature. With a higher temperature, the lower value of MTF is acquired, and thus having a lower lifetime estimation. Figure 2 also shows that the change in cross-sectional area is a large factor in MTF estimation. A larger area will give a longer lifetime due to more material that must be taken away by current carriers. Given that equation (5) derived by using aluminum strips of a large crystallite structure of 8 μ. The properties of how temperature and area effect, the estimation are the same. It is also important to note that the activation energy was a constant given those parameters for aluminum strips. A standard current density is used as a constant for this simulation as well (J = 2 x 105).

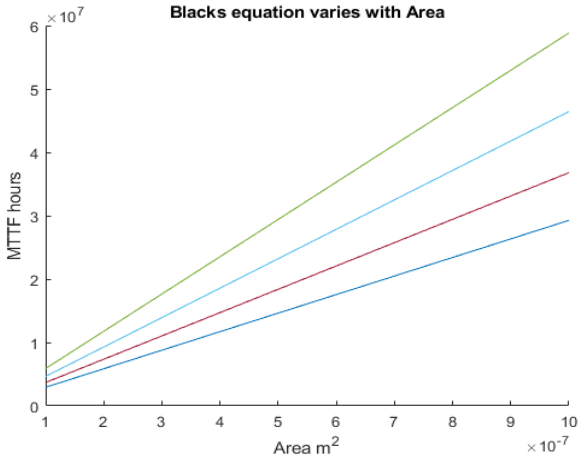

Fig. 2: Black's equation with a variance in area

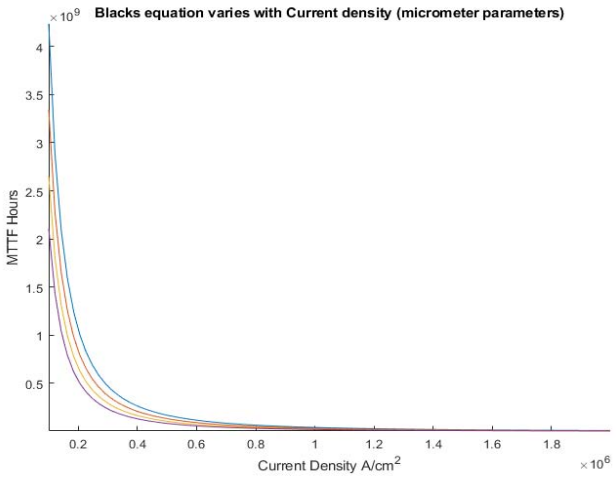

Fig. 3: Current density change in (5) with micrometer parameters

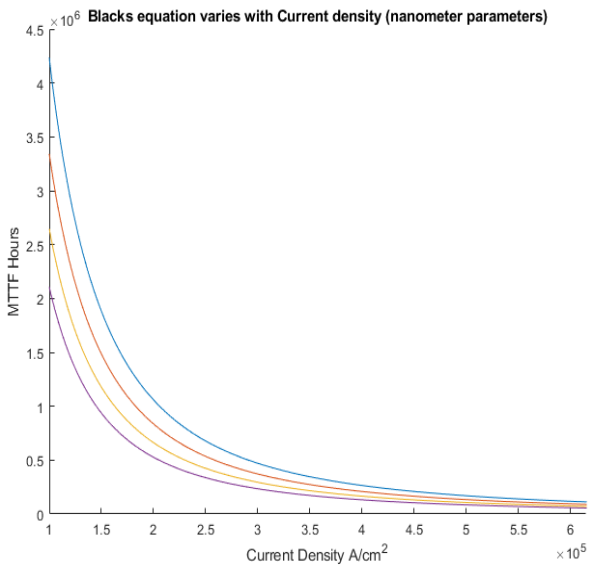

Fig. 4: Change in current density with nanometer parameters

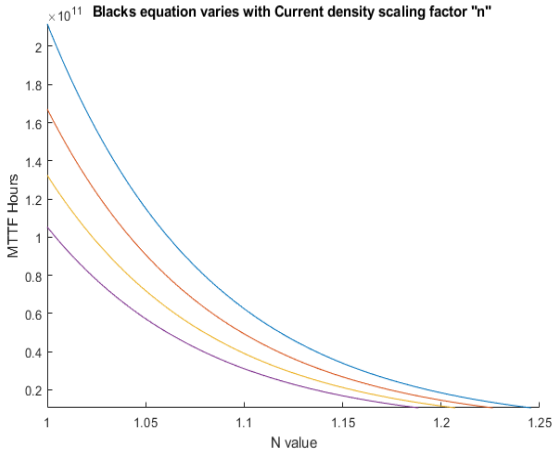

Fig. 5: Represent the behavior of scaling factor (n) that for the current density

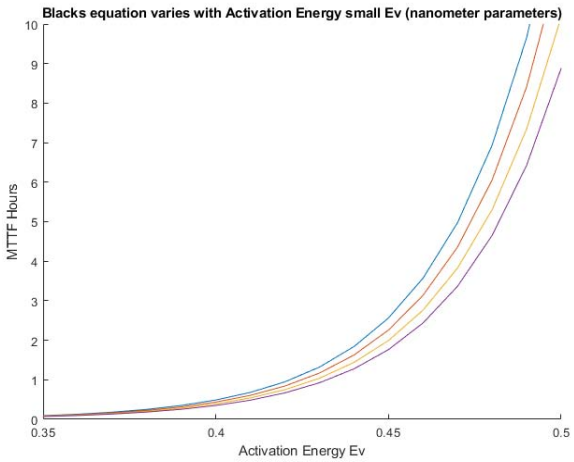

Fig. 6: The relationship between the changes of activation energy to the MTF

The next parameter is in equation (5) that is current density. Simulation is done with both micrometer parameters and nanometer parameters to see if there were any differences. A hypothesis made that the increased amount of current density will have a negative effect on MTF due to current crowding and having a natural increase in temperature. Figure 3 and Figure 4 show that when the current density increased, the MTF reduced drastically. A change in parameters with the area indicates that the MTF will change by a factor of 103 since the formation modeled with equation (5). Temperature is also not as a prominent changing factor in how current density behaves, yet, still follows the same property that temperature has a dependence on MTF. The lower the current density is, the better lifetime an interconnect line will have. Note that when Figure 3 and Figure 4 done, a cross-sectional area had two estimated constants of 180 μm and 180 nm respectively. The same temperature constants used as represented in Figure 2.

As mentioned in the previous works section, the value of n was the center of controversy for a long time.

Figure 5 will show how the value of n changes with the behavior of MTF. The data from Figure 5 shows that the value of n is a critical value as well. Figure 5 shows the value of n between 1 and 1.25. MTF had a change of (20 X 10^{10}). When this data scaled from 1 to 2, the lines are not able to read because of the drastic change in MTF.

The next two figures (Figure 6 and Figure 7) will show a change in MTF with the activation energy variation. Using standard parameters for the cross-sectional area and the scaling factor of current density is 2. Figure 6 shows something more interesting finding. Lower activation energies show an extremely small number for MTF. The activation energy usually found experimentally and it varies from each interconnecting line. Activation energy is the amount of energy that needed for electromigration to begin. Many papers suggest that the activation energy isn't just one value, but a value that changes depending on grain barriers. This simulation was to show the behavior of how the activation energy behaves with the value of MTF. Figure 7 shows a higher activation energy (up to 1) will have an extreme change to MTF. This change happens exponentially. Higher values of an activation energy raises the MTF upwards to 2x108. Values between 6 – 1 will give us a more practical MTF estimation. Knowing this it makes sense that activation energies tend to be in that range depending on what material is used.

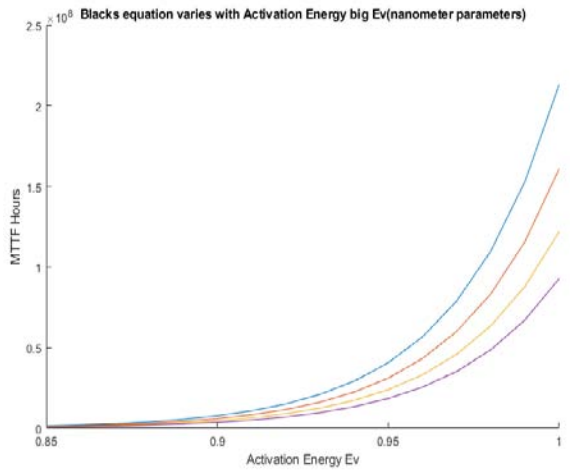

Fig. 7: Effect of the MTTF with respect to the "Big" activation energy

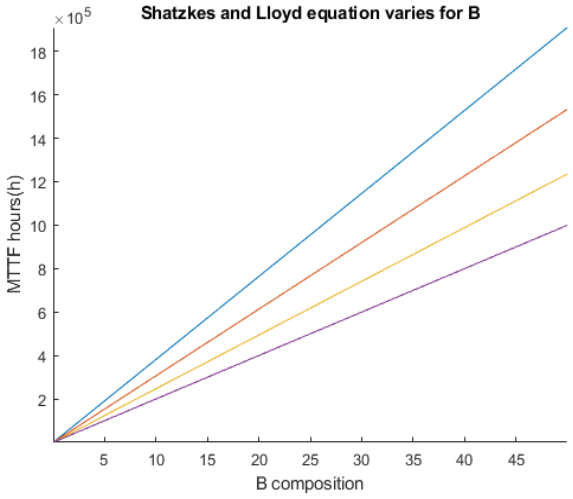

Fig. 8: The behavior of the MTTF with the B composition value

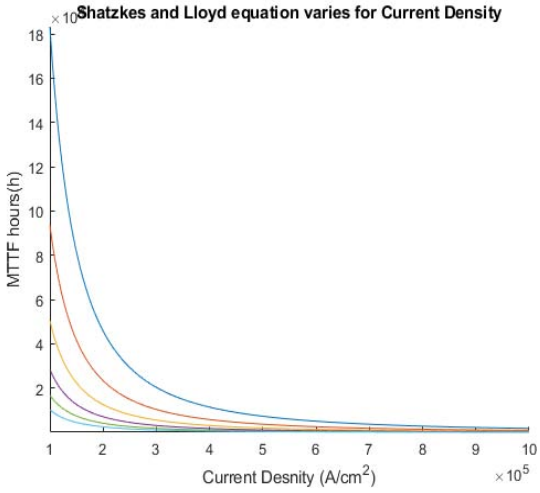

Fig. 9: The Characteristics of MTTF current density with temperature

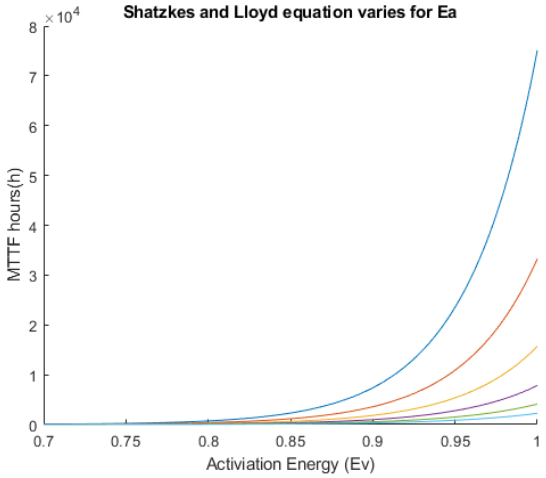

Fig. 10: The relationship between the activation energy and MTTF

Next there are simulations done using equation (3). These followed the same principles of equation (5), and that's with constants being used to show how one variable behaves in the formula. Thus, providing its impact on how MTF evaluated. Figure 8 shows that temperature is still a dependence on MTF. The constants used in the Black equation simulations are the same. With a rise in B's composition, there is a rise in MTF. Figure 9 and 10 show similar characteristics to their respective graphs (Figures 3 and 7). Current density and activation behave the same due to no modification with those parts in (3). Having a low current density will help overall MTF, and a higher activation energy will delay the inevitable electromigration from starting.

Table 1 shows a comparison between equation (1) and equation (3) with the same parameters, and their estimations for MTF with different temperature changes. Changes in temperature are crucial for representing a difference because of the T2 in equation (3). This proves that the changes made in equation (3) is to improve the accuracy of MTF. This also provide a better estimation so equation (1) is much more underestimating technically.

Table 1: Comparing Black's equation with Lloyd and Shatzkes equation

Temperature (K)	MTF from (1)	MTF from (3)
550	2350	2500
560	1712	1825
570	1262	1360
580	939.6	1075
590	706.7	880.4
600	536.6	642.2

IV. Conclusion

This is an investigative work that discuss various process and parameters including temperature impacts on electromigration. This work will give a good idea about electromigration while people will study the electromigration. Although it has to take into account that a lot more parameters other than cross sectional area and temperature involves to analyze the electromigration. Critical grain boundary values need to found. Since VLSI circuits are becoming more and more power demanding and circuits are becoming smaller to support additional functionality in the nanoscale domain, the threat of electromigration continues to grow. This study is done to find what parts of electromigration can be improved in the nano scale domain. Over the course of 50 years, IC's are primarily aluminum and moved on to copper after finding the better electrical and material properties. With different metals, various electromigration could occur. In order to help a small nanoscale copper circuits, a reservoir length is added to a TiN via that are implemented. Simulation work done is to show that temperature has a direct relation on the Mean Time to Failure value. Reducing the temperature for an integrated circuit will improve overall performance. A change is done to Black's equation that is validated to show a better estimation of MTF. The area of electromigration is still being heavily studied in VLSI design and Physics. Refining the equation for MTF is crucial to know the lifetime of an IC in the nanoscale domain.

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Global Journal of Science Frontier Research: a
Physics and Space Science
Volume 18 Issue 9 Version 1.0 Year 2018
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Phenomenon of Kinetic Energy and Inertia of Material Bodies

By F. F. Mende

Abstract- It is well known that for accelerating the material bodies it is necessary to spend energy, in this case the executed work passes into the kinetic energy. With the braking the body returns this energy to the surrounding bodies, for which required the forces, the reverse facts, which accelerated body. This is the phenomenon of the inertia. However, nature of this phenomenon was up to now not clear. In the article it is shown that the inertia and kinetic energy of material bodies are the consequence of the dependence of the scalar potential of charge on the speed. Are obtained the conversions pour on, which reflect this law.

Keywords: *charge, inertia, kinetic energy, scalar-vector potential.*

GJSFR-A Classification: *FOR Code: 030703*

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I. Introduction

It is well known that for accelerating the material bodies it is necessary to spend energy, in this case the executed work passes into the kinetic energy. With the braking the body returns this energy to the surrounding bodies, for which required the forces, the reverse facts, which accelerated body. This is the phenomenon of the inertia.

It is clear that in the process of acceleration the body accumulates some form of energy, which returns then to the environment with its braking. But none of the existing at present theories gives answer to a question, that this after energy and how it is accumulated and returns. Charged the bodies and in the charges are had the electrical field, possessing of the energy. It is possible to expect that the dependence of these pour on from the speed it can shed light to this question. In the special theory of relativity (SR) the electric fields of charges depend on speed, and, it would seem, this theory had to give answer to the presented question. But into SR the charge is the invariant of the speed. Its fields although change in the process of acceleration, these changes occur in such a way that to an increase pour on normal to the direction of motion it is compensated by the decrease of longitudinal pour on, and the flow of the electric field through the surface, which surrounds charge remains constant.

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In works [1-6] it is shown that within the framework the Galileo conversions the scalar potential of charge depends on speed. In this case the electric fields, normal to the direction of its motion, increase, while longitudinal fields they remain constant. Similar of the approach gives of the possibility to explain of the phenomenon of the kinetic energy and the inertia of the material bodies.

II. Kinetic Energy Of The Electrified Bodies

The electron has the electrical fields, energy which easy to calculate. Specific energy of the electrical fields is written

$$w=\frac{1}{2}\varepsilon E^2 .$$

The tension of the electrical fields of the electron is determined by the equality

$$E=\frac{e}{4\pi\varepsilon_0 r^2} .$$

Using the element the volume $4\pi r^2 dr$, we obtain the energy of the fields on resting of the electron:

$$W=\int_a^\infty \frac{e^2 dr}{8\pi\varepsilon_0 r^2}=\frac{e^2}{8\pi\varepsilon_0 a},$$

where e is the charge, a is radius of the electron. If electron moves with the speed v, then, according to the concept of scalar-vector potential, its electric fields, normal to the direction of motion, increase:

$$E_\perp = Ech\frac{v}{c}\approx E\left(1+\frac{1}{2}\frac{v^2}{c^2}\right).$$

Let us write down the electric fields, normal to the direction of motion in the coordinate system, represented in Fig 1.

$$E_\perp = E\left(1+\frac{1}{2}\frac{v^2}{c^2}\right)\sin q$$

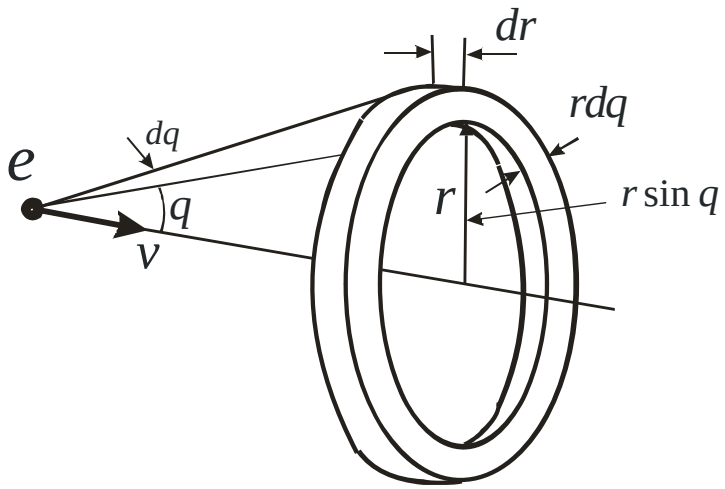

Fig. 1: The element of the volume, utilized for the calculation of the energy fields moving of the electron

Then the energy of the fields moving electron will be written down

$$W_v=\left(1+\frac{1}{2}\frac{v^2}{c^2}\right)^2\int\frac{e^2\sin^3 q\ dqdr}{8\pi\varepsilon_0 r^2}.$$

The integration to the angle gives

$$\int_0^{\pi}\sin^3 q\ dq=-\int_0^{\pi}(1-\cos^2 q)d(\cos q)=-\cos q+\frac{\cos^3 q}{3}=\frac{4}{3}.$$

Therefore

$$W_v=\frac{4}{3}\left(1+\frac{1}{2}\frac{v^2}{c^2}\right)^2\int_a^{\infty}\frac{e^2dr}{8\pi\varepsilon_0 r^2}=\frac{4}{3}\left(1+\frac{v^2}{c^2}+\frac{1}{4}\frac{v^4}{c^4}\right)\frac{e^2}{8\pi\varepsilon_0 a}.$$

For of the speeds is considerable smaller of the speed of the light the term $\frac{1}{4}\frac{v^2}{c^2}$ can be disregarded, therefore

$$W_v=\frac{4}{3}\left(1+\frac{v^2}{c^2}\right)\frac{e^2}{8\pi\varepsilon_0 a}.$$

The connection between by energy of the fields and with the mass of the rest of the electron is given by the equality [7]:

$$W=\frac{4}{3}\frac{e^2}{8\pi\varepsilon_0 a}=mc^2.$$

Consequently, additional energy of electron, connected with the fact that of its field they depend on speed, to be determined by the relationship

$$W_v=mv^2$$

This is the kinetic energy moving of the electron. It is differed from the conventional value in terms of the coefficient $\frac{1}{2}$, but this indicates only that the fact that the officially taken value of the mass of electron must be decreased two times.

Thus, we established the physical cause for the presence in the moving electrified bodies of kinetic energy, and, therefore, also their inertia properties. These the property are connected with the dependence of the scalar potential of charges on the speed, and since all material bodies consist of the free or bound charges, this rule is universal.

III. Conclusion

It is well known that for accelerating the material bodies it is necessary to spend energy, in this case the executed work passes into the kinetic energy. With the braking the body returns this energy to the surrounding bodies, for which be required the forces, the reverse facts, which accelerated body. This is the phenomenon of the inertia. However, nature of this phenomenon was up to now not clear. In the article it is shown that the inertia and kinetic energy of material bodies are the consequence of the dependence of the scalar potential of charge on the speed. The relationships, which reflect this law, are obtained.

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Global Journal of Science Frontier Research (A) Volume XVIII Issue IX Version I Year 2018
18

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Global Journal of Science Frontier Research: a
Physics and Space Science
Volume 18 Issue 9 Version 1.0 Year 2018
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

The Effect of Parametric Amplifier on Entanglement and Statistical Properties of Nondegenerate Three-Level Laser Pumped by Electron Bombardment

By Tamirat Abebe & Tamiru Deressa
Jimma University

Abstract- In this paper, the quantum properties of an electrically pumped non-degenerate three level laser with two mode subharmonic generator coupled to a two-mode vacuum reservoirs via a single-port mirror whose open cavity contains *N* non-degenerate three-level cascade atoms is presented. The analysis is carried out by putting the noise operators associated with a vacuum reservoir in normal order. It is found that the photons as well as atoms of the system are strongly entangled at steady state. It has been shown that the degree of photon entanglement greater than that of atom entanglement. Moreover, the presence of the subharmonic generator leads to an increase in the degree of entanglement.

Keywords: *operator dynamics; quadrature squeezing; entanglement.*

GJSFR-A Classification: *FOR Code: 020399*

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Tamirat Abebe [α] & Tamiru Deressa [σ]

Abstract- In this paper, the quantum properties of an electrically pumped non-degenerate three level laser with two-mode subharmonic generator coupled to a two-mode vacuum reservoirs via a single-port mirror whose open cavity contains *N* non-degenerate three-level cascade atoms is presented. The analysis is carried out by putting the noise operators associated with a vacuum reservoir in normal order. It is found that the photons aswell as atoms of the system are strongly entangled at steady state. It has been shown that the degree of photon entanglement greater than that of atom entanglement. Moreover, the presence of the subharmonic generator leads to an increase in the degree of entanglement.

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I. Introducion

It is now believed that the key ingredient of quantum information is entanglement which has been recognized as the essential resource for quantum teleportation, quantum dense coding, quantum computation, quantum error correction, and quantum cryptography [1-6]. Traditionally, quantum entangled states are considered to be non-classical correlations between individual qubits. However, it has been proved that continuous-variable (CV) entanglement has many advantages in some cases [7, 8]. Recently, much attention has been paid to the generation and detection of CV entanglement as it might be easier to manipulate than the discrete counterparts, quantum bits, in order to perform quantum information processing. On the other hand, the efficiency of quantum information processing highly depends on the degree of entanglement. Hence, it is desirable to generate strongly entangled continuous variable state.

A two-mode subharmonic generator at and above threshold has been theoretically predicted to be a source of light in an entangled state [8,9]. Recently, the experimental realization of the entanglement in two-mode subharmonic generator has been demonstrated by Zhang et al. [10]. Furthermore, Tan et al. [11] have extended the work of Xiong et al. and examined the generation and evolution of the entangled light in the Wigner representation using the sufficient and necessary in separability criteria for a two-mode Gaussian state proposed by Dual et al. [12] and Simon [13]. Tesfa [14] has considered a similar system when the atomic coherence is induced by superposition of atomic states and analyzed the entanglement at steady-state.

Recently, Eyob [15] has studied CV entanglement in a non-degenerate three-level laser with a parametric amplifier. In this model the injected atomic coherence introduced by initially preparing the atoms in a coherent superposition of the top and bottom levels. This combined system exhibits a two-mode squeezed light and produces light in an entangled state. In one model of such a laser, three-level atoms initially in the upper level are injected at a constant rate into the cavity and removed after they have decayed due to spontaneous emission. It appears to be quite difficult to prepare the atoms in a coherent superposition of the top and bottom levels before they are injected into the laser cavity. Besides, it should certainly be hard to find out that the atoms have decayed spontaneously before they are removed from the cavity.

In order to avoid the aforementioned problems, Fesseha [16] have considered that N two-level atoms available in a closed cavity are pumped to the top level by means of electron bombardment. He has shown that the light generated by this laser operating well above threshold is coherent and the light generated by the same laser operating below threshold is chaotic. Moreover, Fesseha [17] has studied the squeezing and the statistical properties of the light produced by a three-level laser with the atoms in a closed cavity and pumped by electron bombardment. He has shown that the maximum quadrature squeezing of the light generated by the laser, operating below threshold, is found to be 50% below the vacuum-state level. In addition, he has also found that the quadrature squeezing of the output light is equal to that of the cavity light. On the other hand, this study shows that the local quadrature squeezing is greater than the global quadrature squeezing. Furthermore, Fesseha [18] has studied the squeezing and the statistical properties of the light produced by a degenerate three-level laser with the atoms in a closed cavity and pumped by coherent light. He has shown that the maximum quadrature squeezing is 43% below the vacuum-state level, which is slightly less than the result found with electron bombardment.

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In this model, we seek to study CV entanglement for the light generated by electrically pumped non-degenerate three-level laser with two-mode subharmonic generator coupled to a two-mode vacuum reservoirs via a single-port mirror whose open cavity contains N non-degenerate threelevel cascade atoms.

In order to carry out our analysis, we put the noise operators associated with the vacuum reservoir in the normal order and we consider the interaction of the three-level atoms with a two mode vacuum reservoir. We then first drive the quantum Langevin equations for the cavity mode operators. We next determine the equations of evolution of the expectation values of atomic operators employing the pertinent master equation. Applying the steady-state solution of equations of evolution, we analyze the mean photon number, CV atomic and photon state entanglement, the quadrature squeezing as well as atom and photon number correlation.

II. Dynamics of Atomic and Cavity Mode Operators

We consider here the case in which non-degenerate three level laser dynamics with two-mode subharmonic generator coupled to two-mode vacuum reservoir whose cavity contains *N* three level atoms in cascade configuration as shown in Fig. 1. We denote the top, intermediate, and bottom levels of these atoms by $|2\rangle_j$, $|1\rangle_j$, and $|0\rangle_j$, respectively. In addition, we assume the cavity mode to be at resonance with the two transitions $|2\rangle_j \leftrightarrow |1\rangle_j$ and $|1\rangle_j \leftrightarrow |0\rangle_j$, with direct transition between levels $|2\rangle_j \leftrightarrow |0\rangle_j$ to be electric dipole forbidden. The pump mode emerging from the two-mode subharmonic generation does not couples to the top and bottom levels. The interaction of a three-level atom with cavity modes and the pump mode with the two-mode subharmonic generation can be described at resonance by the Hamiltonian

$$\hat{H}_S(t) = ig \sum_{n=1,2} \left[\hat{\sigma}_n^{\dagger j}(t)\hat{a}_n(t) - \hat{a}_n^{\dagger}(t)\hat{\sigma}_n^{j}(t)\right] + i\varepsilon(\hat{a}_1^{\dagger}\hat{a}_2^{\dagger} - \hat{a}_1\hat{a}_2), \tag{1}$$

where

$$\hat{\sigma}_1^j(t) = |0\rangle_{jj}\langle 1|, \quad \hat{\sigma}_2^j(t) = |1\rangle_{jj}\langle 2|, \tag{2}$$

are lowering atomic operators, $\hat{a}_1(t)$ and $\hat{a}_2(t)$ are the annihilation operators for the light modes

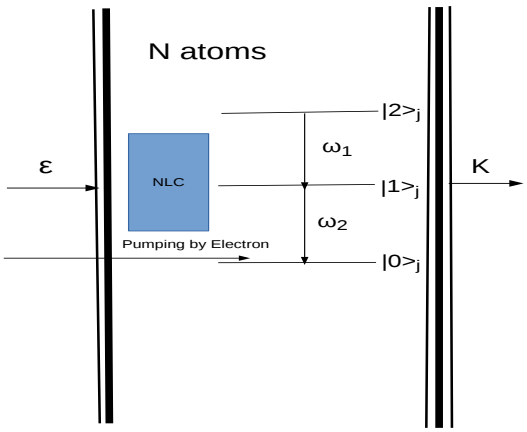

Figure 1: Schematic representation of electrically pumped non-degenerate three level laser dynamics with two-mode subharmonic generator coupled to a two-mode vacuum reservoir

a_1 and a_2, respectively, ε, assumed to be rael and constant, is proportional to the amplitude of the coherent light that drives the NLC, and *g* is the coupling constant between the atom and the cavity modes.

The master equation for a three-level atom coupled to a two-mode vacuum reservoir has the Form

$$\frac{d}{dt}\hat{\rho}(t) = -i[\hat{H}_S, \hat{\rho}] + \frac{\gamma}{2}\sum_{k=0}^{2}\left[2\hat{\sigma}_k^j\hat{\rho}\hat{\sigma}_k^{\dagger j} - \hat{\sigma}_k^{\dagger j}\hat{\sigma}_k^j\hat{\rho} - \hat{\rho}\hat{\sigma}_k^{\dagger j}\hat{\sigma}_k^j\right], \tag{3}$$

in which γ is the spontaneous emission decay constant and $\hat{\sigma}_0^j(t) = |0\rangle_{jj}\langle 2|$. Now with the aid of Eq. (1), one can put (3) in the form

$$\frac{d}{dt}\hat{\rho}(t) = g\sum_{n=1,2}\left[\hat{\sigma}_n^{\dagger j}\hat{a}_n\hat{\rho} - \hat{\rho}\hat{\sigma}_n^{\dagger j}\hat{a}_n + \hat{\rho}\hat{a}_n^{\dagger}\hat{\sigma}_n^{j} - \hat{a}_n^{\dagger}\hat{\sigma}_n^{j}\hat{\rho}\right] + \frac{\gamma}{2}\sum_{k=0}^{2}\left[2\hat{\sigma}_k^{j}\hat{\rho}\hat{\sigma}_k^{\dagger j} - \hat{\sigma}_k^{\dagger j}\hat{\sigma}_k^{j}\hat{\rho} - \hat{\rho}\hat{\sigma}_k^{\dagger j}\hat{\sigma}_k^{j}\right]$$

$$+\varepsilon\left[\hat{a}_1\hat{a}_2\hat{\rho} - \hat{\rho}\hat{a}_1\hat{a}_2 + \hat{\rho}\hat{a}_1^{\dagger}\hat{a}_2^{\dagger} - \hat{a}_1^{\dagger}\hat{a}_2^{\dagger}\hat{\rho}\right]. \tag{4}$$

We model that the laser cavity is coupled to a two-mode vacuum reservoir via a single-port mirror. In addition, we carry out our analysis by putting the noise operators associated with the vacuum reservoir in normal order. Thus the noise operators will not have any effect on the dynamics of the cavity mode operators. We can therefore drop the noise operators and write the quantum Langevin equation for the operators $\hat{a}_n(t)$ as

$$\frac{d}{dt}\hat{a}_n(t) = -i[\hat{a}_n(t), \hat{H}_S(t)] - \frac{1}{2}\kappa\hat{a}_n(t), \tag{5}$$

in which $n = 1, 2$, and κ is assumed to be the cavity damping constant for the light modes a_n. Then with the aid of Eqs. (1) and (5) together with the commutation relation $[\hat{a}_i, \hat{a}_j^{\dagger}] = \delta_{ij}$, we easily find

$$\frac{d}{dt}\hat{a}_n(t) = -g\hat{\sigma}_n^{j} - \frac{1}{2}\kappa\hat{a}_n(t) + \varepsilon\hat{a}_m^{\dagger}, \tag{6}$$

where $n, m = 1, 2$. Now employing the relation $\frac{d}{dt}\langle\hat{A}\rangle = Tr\left(\frac{d\hat{\rho}}{dt}\hat{A}\right)$, along with Eq. (4), one can readily establish that

$$\frac{d}{dt}\langle\hat{\sigma}_1^{j}\rangle = g[\langle\hat{\eta}_0^{j}\hat{a}_1\rangle - \langle\hat{\eta}_1^{j}\hat{a}_1\rangle - \langle\hat{a}_2^{\dagger}\hat{\sigma}_0^{j}\rangle] - \frac{\gamma}{2}\langle\sigma_1^{j}\rangle, \tag{7}$$

$$\frac{d}{dt}\langle\hat{\sigma}_2^{j}\rangle = g[\langle\hat{\eta}_1^{j}\hat{a}_2\rangle - \langle\hat{\eta}_2^{j}\hat{a}_2\rangle + \langle\hat{a}_1^{\dagger}\hat{\sigma}_0^{j}\rangle] - \gamma\langle\sigma_2^{j}\rangle, \tag{8}$$

$$\frac{d}{dt}\langle\hat{\sigma}_0^{j}\rangle = g[\langle\hat{\sigma}_1^{j}\hat{a}_2\rangle - \langle\hat{\sigma}_2^{j}\hat{a}_1\rangle] - \frac{\gamma}{2}\langle\sigma_0^{j}\rangle, \tag{9}$$

$$\frac{d}{dt}\langle\hat{\eta}_1^{j}\rangle = g[\langle\hat{\sigma}_1^{\dagger j}\hat{a}_1\rangle + \langle\hat{a}_1^{\dagger}\hat{\sigma}_1^{j}\rangle - \langle\hat{\sigma}_2^{\dagger j}\hat{a}_2\rangle - \langle\hat{a}_2^{\dagger}\hat{\sigma}_2^{j}\rangle] + \gamma(\langle\eta_2^{j}\rangle - \langle\eta_1^{j}\rangle), \tag{10}$$

$$\frac{d}{dt}\langle\hat{\eta}_2^{j}\rangle = g[\langle\hat{\sigma}_2^{\dagger j}\hat{a}_2\rangle + \langle\hat{a}_2^{\dagger}\hat{\sigma}_2^{j}\rangle] - 2\gamma\langle\eta_2^{j}\rangle, \tag{11}$$

$$\frac{d}{dt}\langle\hat{\eta}_0^{j}\rangle = -g[\langle\hat{\sigma}_1^{\dagger j}\hat{a}_1\rangle + \langle\hat{a}_1^{\dagger}\hat{\sigma}_1^{j}\rangle] + \gamma(\langle\eta_1^{j}\rangle + \langle\eta_2^{j}\rangle) \tag{12}$$

where $\hat{\eta}_k^{j}(t) = |k\rangle_{jj}\langle k|$ and $k = 0, 1, 2$.

The three-level atoms available in the cavity are pumped from the bottom level to the top level by means of electron bombardment. The pumping process must surely affect the dynamics of $\langle\hat{\eta}_2^{j}\rangle$ and $\langle\hat{\eta}_0^{j}\rangle$ If τ_a a represents the rate at which a single atom is pumped from the bottom to the top level, then $\langle\hat{\eta}_2^{j}\rangle$ increases at the rate of $\tau_a\langle\hat{\eta}_0^{j}\rangle$ and decreases at the the same rate. We see that the equations in (7)-(12) are coupled nonlinear differential equations and hence it is not possible to find exact time-dependent solutions of these equations. We intend to overcome this problem by applying the large-time approximation. Then employing this approximation scheme, we get from Eq. (6) and, the approximately valid relation

$$\hat{a}_n = \frac{2\varepsilon}{\kappa}\hat{a}_m^{\dagger} - \frac{2g}{\kappa}\hat{\sigma}_n^{j}(t), \tag{13}$$

where $m, n = 1, 2$. Evidently, these turn out to be exact relations at steady-state. It then follows that

$$\hat{a}_n = -\frac{4\varepsilon g}{\kappa^2 - 4\varepsilon^2}\hat{\sigma}_m^{\dagger j} - \frac{2g\kappa}{\kappa^2 - 4\varepsilon^2}\hat{\sigma}_n^{j}. \tag{14}$$

Now introducing Eq. (14) into Eqs. (7)-(12) and summing over the N three-level atoms, we see that

$$\frac{d}{dt}\langle\hat{\Sigma}_1\rangle = -\frac{R}{2}\langle\hat{\Sigma}_1\rangle, \tag{15}$$

$$\frac{d}{dt}\langle\hat{\Sigma}_2\rangle = -\frac{R}{2}\langle\hat{\Sigma}_2\rangle - T\langle\hat{\Sigma}_1^{\dagger}\rangle, \tag{16}$$

$$\frac{d}{dt}\langle\hat{\Sigma}_0\rangle = -\frac{R}{2}\langle\hat{\Sigma}_0\rangle + T[\langle\hat{N}_1\rangle - \langle\hat{N}_0\rangle], \tag{17}$$

$$\frac{d}{dt}\langle\hat{N}_1\rangle = -R[\langle\hat{N}_1\rangle - \langle\hat{N}_2\rangle] + T(\langle\hat{\Sigma}_0\rangle + \langle\hat{\Sigma}_0^{\dagger}\rangle) \tag{18}$$

$$\frac{d}{dt}\langle\hat{N}_2\rangle = -R\langle\hat{N}_2\rangle - T(\langle\hat{\Sigma}_0\rangle + \langle\hat{\Sigma}_0^{\dagger}\rangle) + \tau_a\langle\hat{N}_0\rangle, \tag{19}$$

$$\frac{d}{dt}\langle\hat{N}_0\rangle = -\tau_a\langle\hat{N}_0\rangle + R\langle\hat{N}_1\rangle + \gamma\langle\hat{N}_2\rangle, \tag{20}$$

Where

$$\gamma_c = \frac{4g^2}{\kappa}, \tag{21}$$

$$R = \gamma + \frac{\gamma_c\kappa^2}{\kappa^2 - 4\varepsilon^2}, \tag{22}$$

$$T = \frac{\gamma_c\varepsilon\kappa}{\kappa^2 - 4\varepsilon^2}, \tag{23}$$

$\hat{\Sigma}_1 = \sum_{j=1}^{N}\hat{\sigma}_1^j$, $\hat{\Sigma}_2 = \sum_{j=1}^{N}\hat{\sigma}_2^j$, $\hat{\Sigma}_0 = \sum_{j=1}^{N}\hat{\sigma}_0^j$, $\hat{N}_0 = \sum_{j=1}^{N}\hat{\eta}_0^j$, $\hat{N}_1 = \sum_{j=1}^{N}\hat{\eta}_1^j$, $\hat{N}_2 = \sum_{j=1}^{N}\hat{\eta}_2^j$, with the operators $\hat{N}_2$, $\hat{N}_1$, and $\hat{N}_0$ representing the number of atoms in the top, middle, and bottom levels. We prefer to call the parameter defined by Eq. (21) the stimulated emission decay constant. In addition, employing the completeness relation

$$\hat{I} = \hat{\eta}_0^j + \hat{\eta}_1^j + \hat{\eta}_2^j, \tag{24}$$

we easily arrive at

$$N = \langle\hat{N}_0\rangle + \langle\hat{N}_1\rangle + \langle\hat{N}_2\rangle. \tag{25}$$

Furthermore, applying the definition given by Eq. (2) and setting for any j

$$\hat{\sigma}_1^j = |0\rangle\langle 1|, \tag{26}$$

we have

$$\hat{\Sigma}_1 = N|0\rangle\langle 1|. \tag{27}$$

Following the same procedure, one can also check that $\hat{\Sigma}_2 = N|1\rangle\langle 2|$, $\hat{\Sigma}_0 = N|0\rangle\langle 2|$, $\hat{N}_0 = N|0\rangle\langle 0|$, $\hat{N}_1 = N|1\rangle\langle 1|$, and $\hat{N}_2 = N|2\rangle\langle 2|$. Moreover, using the definition

$$\hat{\Sigma} = \hat{\Sigma}_1 + \hat{\Sigma}_2 \tag{28}$$

and taking into account the above results, it can be readily established that

$$\hat{\Sigma}^{\dagger}\hat{\Sigma} = N(\hat{N}_1 + \hat{N}_2), \quad \hat{\Sigma}\hat{\Sigma}^{\dagger} = N(\hat{N}_1 + \hat{N}_0), \hat{\Sigma}^2 = N\hat{\Sigma}_0. \tag{29}$$

Upon adding the two separate equations from Eq. (6), we have

$$\frac{d}{dt}\hat{a}(t) = -g\hat{\sigma}^j - \frac{1}{2}\kappa\hat{a}(t) + \varepsilon\hat{a}^{\dagger}, \tag{30}$$

in which

$$\hat{a}(t) = \hat{a}_1(t) + \hat{a}_2(t), \tag{31}$$

$$\hat{\sigma}^j = \hat{\sigma}_1^j + \hat{\sigma}_2^j. \tag{32}$$

Applying large-time approximation scheme into Eq. (20), we see that

$$\hat{a} = -\frac{2g\kappa}{\kappa^2 - 4\varepsilon^2}\hat{\sigma}^j - \frac{4g\varepsilon}{\kappa^2 - 4\varepsilon^2}\hat{\sigma}^{\dagger j}(t). \tag{33}$$

Taking into account of Eq. (33) and its complex conjugate and on summing over all atoms, the commutation relation of the cavity mode operator is

$$[\hat{a}, \hat{a}^{\dagger}] = \frac{\gamma_c\kappa}{\kappa^2 - 4\varepsilon^2}(\hat{N}_0 - \hat{N}_2), \tag{34}$$

where $[\hat{a}, \hat{a}^{\dagger}] = \sum_{\neq 1}^{N}[\hat{a}, \hat{a}^{\dagger}]_j$, stands for the commutator of $\hat{a}$ and $\hat{a}^{\dagger}$ when the cavity mode is interacting with all the N three-level atoms. In the presence of N three-level atoms, we rewrite Eq. (30) as

$$\frac{d}{dt}\hat{a}(t) = -\frac{1}{2}\Big(\frac{\kappa^2 - 4\varepsilon^2}{\kappa}\Big)\hat{a}(t) + \lambda_1\hat{\Sigma} + \lambda_2\hat{\Sigma}^{\dagger}, \quad (35)$$

in which λ_1 and λ_2 is a constant whose values remains to be fixed. The steady-state solution of Eq. (35) is

$$\hat{a} = \frac{2\lambda_1\kappa}{\kappa^2 - 4\varepsilon^2}\hat{\Sigma} + \frac{2\lambda_2\kappa}{\kappa^2 - 4\varepsilon^2}\hat{\Sigma}^{\dagger}(t). \quad (36)$$

In view of (36) and its complex conjugate, the commutation relation for the cavity mode operator Is

$$[\hat{a}, \hat{a}^{\dagger}] = \frac{4\kappa^2(\lambda_1^2 - \lambda_2^2)}{(\kappa^2 - 4\varepsilon^2)^2}N(\hat{N}_0 - \hat{N}_2). \quad (37)$$

Comparing Eqs. (34) and (37) to solve λ_1 and λ_2, consider the case λ_1 - $\lambda_2 \neq 0$, then we see that

$$\lambda_1 = \frac{g}{\sqrt{N}}, \quad and \quad \lambda_2 = \frac{2g\varepsilon}{\kappa\sqrt{N}}. \quad (38)$$

Then Eq. (35) can be rewritten as

$$\hat{a} = \frac{2g\kappa}{\sqrt{N}(\kappa^2 - 4\varepsilon^2)}\hat{\Sigma} + \frac{4g\varepsilon}{\sqrt{N}(\kappa^2 - 4\varepsilon^2)}\hat{\Sigma}^{\dagger}. \quad (39)$$

Now from Eq. (15), one can write

$$\frac{d}{dt}\langle\hat{\Sigma}_1\rangle = -\frac{1}{2}\xi\langle\hat{\Sigma}_1\rangle, \quad (40)$$

where $\xi = \frac{A}{\tau_a C}$. We notice that the steady-state solution of $\langle\hat{\Sigma}_1\rangle$ of (10) for ξ different from zero is

$$\langle\hat{\Sigma}_1\rangle = 0. \quad (41)$$

Similarly,

$$\langle\hat{\Sigma}_2\rangle = \langle\hat{\Sigma}\rangle = 0. \quad (42)$$

With the aid of (42) and the assumption that the cavity light is initially in a vacuum state, the expectation value of the solution of (35) is found to be

$$\langle\hat{a}(t)\rangle = 0. \quad (43)$$

We observe on the basis of Equations (35) and (43) that $\hat{a}$ is a Gaussian variable with zero mean. We next seek to calculate the expectation value of the operators $\hat{N}_0, \hat{N}_1, \hat{N}_2$ and the atomic operator $\hat{\Sigma}_0$. To this end, applying the large time approximation scheme to Eqs. (15)-(20) along with (25), we readily find

$$\langle\hat{N}_2\rangle = \frac{R\tau_a(R^2 + \gamma\tau_a) - 4T^2(R - \tau_a)(R + \gamma)}{R^2(\tau_a + \gamma)(R + \tau_a)}N, \quad (44)$$

$$\langle\hat{N}_1\rangle = \frac{R}{R + \tau_a}N, \quad (45)$$

$$\langle\hat{N}_0\rangle = \frac{R^2 + \tau_a\gamma}{(\tau_a + \gamma)(R + \tau_a)}N, \quad (46)$$

$$\langle\hat{\Sigma}_0\rangle = \frac{2T(R + \gamma)(R - \tau_a)}{R(\tau_a + \gamma)(R + \tau_a)}N. \quad (47)$$

III. The Mean Photon Number

We now proceed to obtain the mean photon number of light mode a in the entire frequency interval. The mean photon number of light mode a, represented by the operators $\hat{a}$ and $\hat{a}^{\dagger}$, is defined by

$$\overline{n} = \langle\hat{a}^{\dagger}\hat{a}\rangle. \quad (48)$$

With the aid of Eq. (39) and its complex conjugate, the mean photon number is expressible as

$$\overline{n} = \frac{\gamma_c\kappa}{(\kappa^2 - 4\varepsilon^2)^2}\left[\kappa^2\Big(\langle\hat{N}_1\rangle + \langle\hat{N}_2\rangle\Big) + 4\varepsilon^2\Big(N - \langle\hat{N}_2\rangle\Big) + 4\kappa\varepsilon\langle\hat{\Sigma}_0\rangle\right]. \quad (49)$$

For the case in which $\varepsilon = 0$, (49) turns out to be

$$\overline{n} = \frac{\gamma_c}{\kappa}\left[\frac{(\gamma + \gamma_c)^2(\gamma + 2\tau_a) + \gamma\tau_a^2}{(\gamma + \gamma_c)(\tau_a + \gamma)(\gamma + \gamma_c + \tau_a)}\right]N. \quad (50)$$

In the absence of spontaneous emission when $(\gamma = 0)$, Eq. (50) reduces to

$$\overline{n} = \frac{2\gamma_c^2}{\kappa(\gamma_c + \tau_a)}N. \quad (51)$$

It proves to be convenient to refer to the regime of laser operation with more atoms in the top

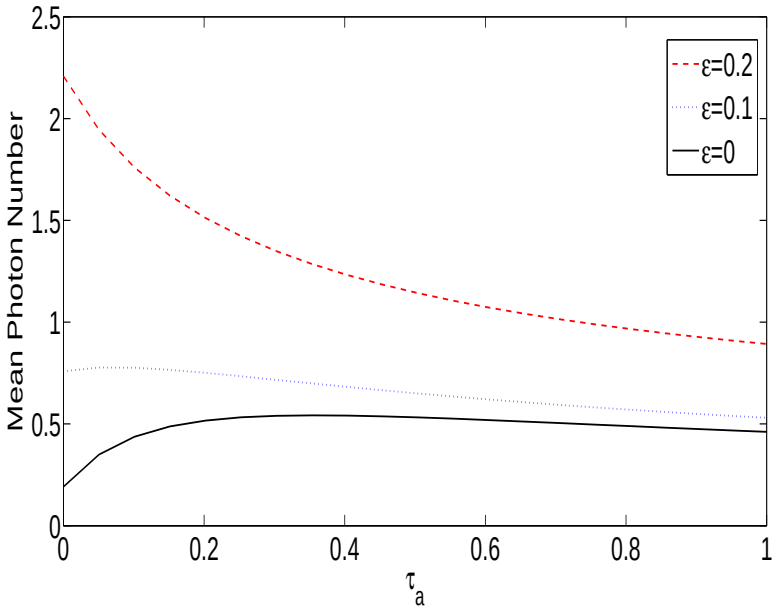

Figure 2: Plots of the mean photon number [Eq. (49)] versus τ_a for $\gamma_c = 0.4$, $\kappa = 0.8$, $N = 50$, $\gamma = 0.2$ and for different values of ε.

level than in the bottom level as above threshold, the regime of laser operation with equal number of atoms in the top and bottom levels as threshold, and the regime of laser operation with less atoms in the top level than in the bottom level as below threshold. Thus according to Eq. (51) for the laser operating above threshold $\gamma_c < \tau_a$, for the laser operating at threshold $\gamma_c = \tau_a$, and for the laser operating below threshold $\gamma_c > \tau_a$. We note that for well above threshold

$$\bar{n} = \frac{2\gamma_c}{\kappa\tau_a}N,$$

for below threshold

$$\bar{n} = \frac{2\gamma_c}{\kappa}N,$$

and at threshold

$$\bar{n} = \frac{\gamma_c}{\kappa}N.$$

IV. Photon-Number Correlation and Atom-Number Correlation

In this section we seek to analyze the degree of photon-number correlation and atom-number correlation. In the cascading transition from energy level $|2\rangle_j$ to $|0\rangle_j$ via $|1\rangle_j$, a correlation between the two emitted photons a_1 and a_2 can readily be established. Hence the photon number correlation for the cavity modes can be defined as

$$g(\hat{n}_1, \hat{n}_2) = \frac{\langle \hat{a}_1^\dagger(t)\hat{a}_1(t)\hat{a}_2^\dagger(t)\hat{a}_2(t)\rangle}{\langle \hat{a}_1^\dagger(t)\hat{a}_1(t)\rangle\langle \hat{a}_2^\dagger(t)\hat{a}_2(t)\rangle}. \tag{52}$$

On the other hand, using Eq. (6) together with (14), the equation of evolution of cavity mode operators can be rewritten as

$$\frac{d}{dt}\hat{a}_n(t) = -\frac{1}{2}\left(\frac{\kappa^2 - 4\varepsilon^2}{\kappa}\right)\hat{a}_n(t) - g\hat{\sigma}_n^j - \frac{2g\varepsilon}{\kappa}\hat{\sigma}_m^{\dagger j}, \tag{53}$$

where $n, m = 1, 2$. Applying the steady-state solution of Eq. (53), one can readily establish that the commutation relation of the cavity mode operators $\hat{a}_1$ and $\hat{a}_1^\dagger$ as well as $\hat{a}_2$ and $\hat{a}_2^\dagger$ with summing over all atoms. We then notice that

$$[\hat{a}_1, \hat{a}_1^\dagger] = \frac{\gamma_c\kappa}{(\kappa^2 - 4\varepsilon^2)^2}\left[4\varepsilon^2\left(\hat{N}_2 - \hat{N}_1\right) + \kappa^2\left(\hat{N}_0 - \hat{N}_1\right) + 2\kappa\varepsilon\left(\hat{\Sigma}_0 + \hat{\Sigma}_0^\dagger\right)\right] \tag{54}$$

and

$$[\hat{a}_2, \hat{a}_2^\dagger] = \frac{\gamma_c \kappa}{(\kappa^2 - 4\varepsilon^2)^2}\left[4\varepsilon^2\left(\hat{N}_1 - \hat{N}_0\right) + \kappa^2\left(\hat{N}_1 - \hat{N}_2\right) - 2\kappa\varepsilon\left(\hat{\Sigma}_0 + \hat{\Sigma}_0^\dagger\right)\right], \tag{55}$$

where

$$[\hat{a}_i, \hat{a}_k^\dagger] = \delta_{ik}\sum_{j=1}^{N}[\hat{a}_i, \hat{a}_k^\dagger]_j, \tag{56}$$

stands for the commutator of mode operators when the cavity light is interacting with all the *N* three-level atoms. In the presence of *N* three-level atoms, we rewrite Eq. (52) as

$$\frac{d}{dt}\hat{a}_n(t) = -\frac{1}{2}\left(\frac{\kappa^2 - 4\varepsilon^2}{\kappa}\right)\hat{a}_n(t) + \lambda_n'\hat{\Sigma}_n + \lambda_n''\hat{\Sigma}_m^\dagger, \tag{57}$$

in which λ_n', λ_n'', are constants whose values remain to be fixed. The steady-state solution of Eq. (57) is

$$\hat{a}_n = \frac{2\lambda_n'\kappa}{\kappa^2 - 4\varepsilon^2}\hat{\Sigma}_n + \frac{2\lambda_n''\kappa}{\kappa^2 - 4\varepsilon^2}\hat{\Sigma}_m^\dagger, \tag{58}$$

where $n, m = 1, 2$. In view of (58) as well as its complex conjugate, the commutation relation for the cavity mode operators is

$$[\hat{a}_1, \hat{a}_1^\dagger] = \frac{4N\kappa^2}{(\kappa^2 - 4\varepsilon^2)^2}\left[\lambda_1''^2(\hat{N}_2 - \hat{N}_1) + \lambda_1'^2(\hat{N}_0 - \hat{N}_1) + \lambda_1'\lambda_1''(\hat{\Sigma}_0 + \hat{\Sigma}_0^\dagger)\right] \tag{59}$$

and

$$[\hat{a}_2, \hat{a}_2^\dagger] = \frac{4N\kappa^2}{(\kappa^2 - 4\varepsilon^2)^2}\left[\lambda_2''^2(\hat{N}_1 - \hat{N}_0) + \lambda_2'^2(\hat{N}_1 - \hat{N}_2) - \lambda_2'\lambda_2''(\hat{\Sigma}_0 + \hat{\Sigma}_0^\dagger)\right]. \tag{60}$$

On comparing Eqs. (54) and (59) together with (55) and (60), we obtain $\lambda_1' = \lambda_2' = \lambda_1$ and $\lambda_1'' = \lambda_2'' = \lambda_2$. Hence Eq. (58) can be rewritten as

$$\hat{a}_n = \frac{2g\kappa}{\sqrt{N}(\kappa^2 - 4\varepsilon^2)}\hat{\Sigma}_n + \frac{4g\varepsilon}{\sqrt{N}(\kappa^2 - 4\varepsilon^2)}\hat{\Sigma}_m^\dagger. \tag{61}$$

Now in view of Eqs. (41) and (42) as well as the assumption that the cavity light is initially in a vacuum state, the expectation value of the solutions of Eq. (57) is found to be

$$\langle \hat{a}_n(t)\rangle = 0. \tag{62}$$

On account of Eq. (62) together with Eq. (57) that $\hat{a}_n(t)$ is a Gaussian variables with zero mean. Thus employing these results, the photon-number correlation turns out to be

$$g(\hat{n}_1, \hat{n}_2)_p = 1 + \frac{4\varepsilon\kappa\left[\left(\kappa^2 + 4\varepsilon^2\right)\langle\hat{\Sigma}_0\rangle + 2\kappa\varepsilon\left(\langle\hat{N}_2\rangle + \langle\hat{N}_0\rangle\right)\right]}{\left[\kappa^2 + 4\varepsilon^2\right]\left[\kappa^2\langle\hat{N}_2\rangle + 4\varepsilon\kappa\langle\hat{\Sigma}_0\rangle + 4\varepsilon^2\langle\hat{N}_0\rangle\right]}. \tag{63}$$

On the other hand, the atom-number correlation is defined by

$$g(\hat{n}_1, \hat{n}_2)_a = \frac{\langle\hat{\Sigma}_1^\dagger\hat{\Sigma}_1\hat{\Sigma}_2^\dagger\hat{\Sigma}_2\rangle}{\langle\hat{\Sigma}_1^\dagger\hat{\Sigma}_1\rangle\langle\hat{\Sigma}_2^\dagger\hat{\Sigma}_2\rangle}. \tag{64}$$

We recall that the atomic operators $\hat{\Sigma}_1$ and $\hat{\Sigma}_2$ are Gaussian variables with zero mean. Hence Eq. (64) can be rewritten as

$$g(\hat{n}_1,\hat{n}_2)_a = 1 + \frac{\langle\hat{\Sigma}_1^\dagger\hat{\Sigma}_2^\dagger\rangle\langle\hat{\Sigma}_1\hat{\Sigma}_2\rangle + \langle\hat{\Sigma}_1^\dagger\hat{\Sigma}_2\rangle\langle\hat{\Sigma}_1\hat{\Sigma}_2^\dagger\rangle}{\langle\hat{\Sigma}_1^\dagger\hat{\Sigma}_1\rangle\langle\hat{\Sigma}_2^\dagger\hat{\Sigma}_2\rangle}. \tag{65}$$

It then follows that

$$g(\hat{n}_1,\hat{n}_2)_a = 1. \tag{66}$$

We immediately see that the maximum degree of photon number correlation observed when more atoms in the lower energy level than on the upper level. This occurs when the three-level laser is operating below threshold. On the other hand, we note that from Eqs. (63) and (66) that unlike the photon-number correlation, the atoms in the laser cavity are not correlated. Moreover, we point out that in the absence of subharmonic generator, one can never realize correlated photons in the laser cavity.

V. Quadrature Squeezing

In this section, we wish to calculate the quadrature squeezing of the cavity light in the entire frequency interval. The squeezing properties of the cavity light are described by two quadrature operators

$$\hat{a}_+ = \hat{a}^\dagger + \hat{a}, \qquad \hat{a}_- = i(\hat{a}^\dagger - \hat{a}). \tag{67}$$

It can be readily established that

$$[\hat{a}_+,\hat{a}_-] = \left[\frac{2i\gamma_c\kappa}{\kappa^2-4\varepsilon^2}\right]\left[\langle\hat{N}_2\rangle - \langle\hat{N}_0\rangle\right]. \tag{68}$$

It then follows that

$$\Delta a_+\Delta a_- \geq \left[\frac{\gamma_c\kappa}{\kappa^2-4\varepsilon^2}\right]\left[\langle\hat{N}_2\rangle - \langle\hat{N}_0\rangle\right]. \tag{69}$$

The variance of the quadrature operators is expressible as

$$(\Delta a_\pm)^2 = \langle\hat{a}^\dagger\hat{a}\rangle + \langle\hat{a}\hat{a}^\dagger\rangle \pm [\langle\hat{a}^2\rangle + \langle\hat{a}^{\dagger 2}\rangle] \mp [\langle\hat{a}\rangle^2 + \langle\hat{a}^\dagger\rangle^2] - 2\langle\hat{a}\rangle\langle\hat{a}^\dagger\rangle. \tag{70}$$

Since $\hat{a}$ is a Gaussian variable with zero mean, the quadrature variance turns out to be

$$(\Delta a_\pm)^2 = \langle\hat{a}^\dagger\hat{a}\rangle + \langle\hat{a}\hat{a}^\dagger\rangle \pm [\langle\hat{a}^2\rangle + \langle\hat{a}^{\dagger 2}\rangle]. \tag{71}$$

It then follows that

$$(\Delta a_\pm)^2 = \frac{\gamma_c\kappa}{(\kappa\mp 2\varepsilon)^2}[N + \langle\hat{N}_1\rangle \pm 2\langle\hat{\Sigma}_0\rangle]. \tag{72}$$

We recall that the light generated by a two-level laser operating well above threshold is coherent, the quadrature variance of which is given by [12]

$$(\Delta a_\pm)_c^2 = \frac{\gamma_c}{\kappa}N. \tag{73}$$

We calculate the quadrature squeezing of the cavity light relative to the quadrature variance of the cavity coherent light. We then define the quadrature squeezing of the cavity light by

$$S = \frac{(\Delta a_\pm)_c^2 - (\Delta a_-)^2}{(\Delta a_\pm)_c^2}. \tag{74}$$

Hence employing (72) and (73), one can put Eq. (74) in the form

$$S = 1 - \frac{\kappa^2[N + \langle\hat{N}_1\rangle - 2\langle\hat{\Sigma}_0\rangle]}{N(\kappa+2\varepsilon)^2}. \tag{75}$$

We observe that, unlike the mean photon number, the quadrature squeezing does not depend on the number of atoms. This implies that the quadrature squeezing of the cavity light is independent of the number of photons.

VI. Entanglement

To this end, we prefer to analyze the entanglement of photon-states in the laser cavity. Quantum entanglement is a physical phenomenon that occurs when pairs or groups of particles cannot be described independently instead, a quantum state may be given for the system as a whole. Measurements of physical properties such as position, momentum, spin, polarization, etc. performed on entangled particles are found to be appropriately correlated. A pair of particles is taken to be entangled in quantum theory, if its states cannot be expressed as a product of the states of its individual constituents. The preparation and manipulation of these entangled states that have nonclassical and nonlocal properties lead to a better

understanding of the basic quantum principles. It is in this spirit that this section is devoted to the analysis of the entanglement of the two-mode photon states. In other words, it is a well-known fact that a quantum system is said to be entangled, if it is not separable. That is, if the density operator for the combined state cannot be described as a combination of the product density operators of the constituents,

$$\hat{\rho} \neq \sum_k p_k \hat{\rho}_k^{(1)} \otimes \hat{\rho}_k^{(2)}, \tag{76}$$

in which $p_k \gg 0$ and $\sum_k p_k = 1$ to verify the normalization of the combined density states. On the other hand, a maximally entangled CV state can be expressed as a coeigenstate of a pair of EPR-type operators [19] such as $\hat{X}_2 - \hat{X}_1$ and $\hat{P}_2 - \hat{P}_1$. The total variance of these two operators reduces to zero for maximally entangled CV states. According to the criteria given by Duan et al [12], cavity photons of a system are entangled, if the sum of the variance of a pair of EPR-like operators,

$$\hat{s} = \hat{X}_2 - \hat{X}_1, \tag{77}$$

$$\hat{t} = \hat{P}_2 + \hat{P}_1, \tag{78}$$

where $\hat{X}_1 = \frac{1}{\sqrt{2}}(\hat{a}_1 + \hat{a}_1^\dagger)$, $\hat{X}_2 = \frac{1}{\sqrt{2}}(\hat{a}_2 + \hat{a}_2^\dagger)$, $\hat{P}_1 = \frac{i}{\sqrt{2}}(\hat{a}_1^\dagger - \hat{a}_1)$, and $\hat{P}_2 = \frac{i}{\sqrt{2}}(\hat{a}_2^\dagger - \hat{a}_2)$ are quadrature operators for modes a_1 and a_2, satisfy

$$(\Delta s)^2 + (\Delta t)^2 < 2N \tag{79}$$

and recalling that the cavity mode operators $\hat{a}_1$ and $\hat{a}_2$ are Gaussian variables with zero mean, we readily get

$$(\Delta s)^2 + (\Delta t)^2 = \left[\langle \hat{a}_1^\dagger \hat{a}_1 \rangle + \langle \hat{a}_1 \hat{a}_1^\dagger \rangle + \langle \hat{a}_2^\dagger \hat{a}_2 \rangle + \langle \hat{a}_2 \hat{a}_2^\dagger \rangle\right] - \left[\langle \hat{a}_1 \hat{a}_2 \rangle + \langle \hat{a}_1^\dagger \hat{a}_2^\dagger \rangle + \langle \hat{a}_2 \hat{a}_1 \rangle + \langle \hat{a}_2^\dagger \hat{a}_1^\dagger \rangle\right]. \tag{80}$$

It then follows that

$$(\Delta s)^2 + (\Delta t)^2 = 2(\Delta a_-)^2. \tag{81}$$

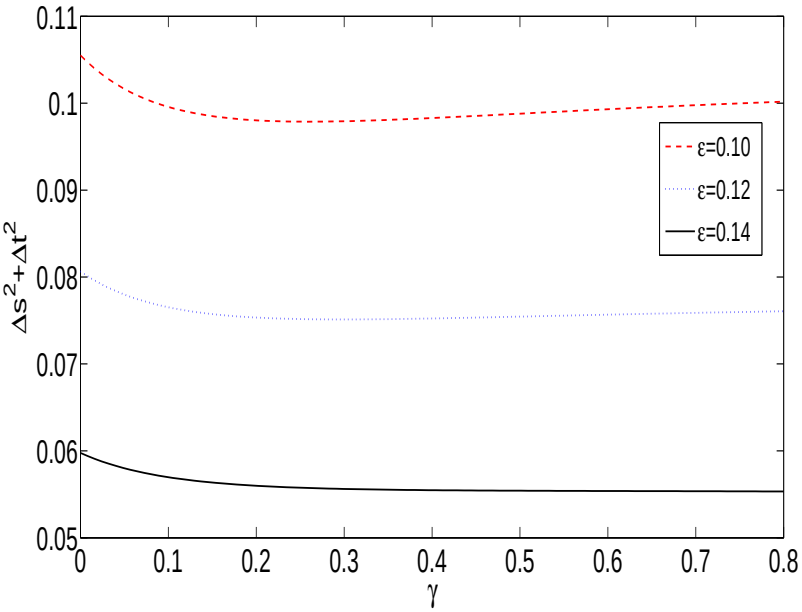

Figure 3: Plots of photon entanglement [Eq. (81)] versus γ for $\gamma_c = 0.4, \kappa = 0.8, N = 50, \tau_a = 0.2$, and for different values of ε.

In the absence of subharmonic generator, Eq. (81) takes the form

$$(\Delta s)^2 + (\Delta t)^2 = \left[\frac{\gamma_c}{\kappa} N\right]\left[\frac{2\gamma_c + 2\gamma + \tau_a}{\gamma + \gamma_c + \tau_a}\right] \tag{82}$$

We observe from Eq. (82) that as the stimulated decay constant increases, the degree of entanglement increases. On the basis of the criteria (79), we clearly see that the two states of the generated light are strongly entangled at steady-state. Moreover, the

presence of the subharmonic generator leads to an increase in the degree of entanglement.

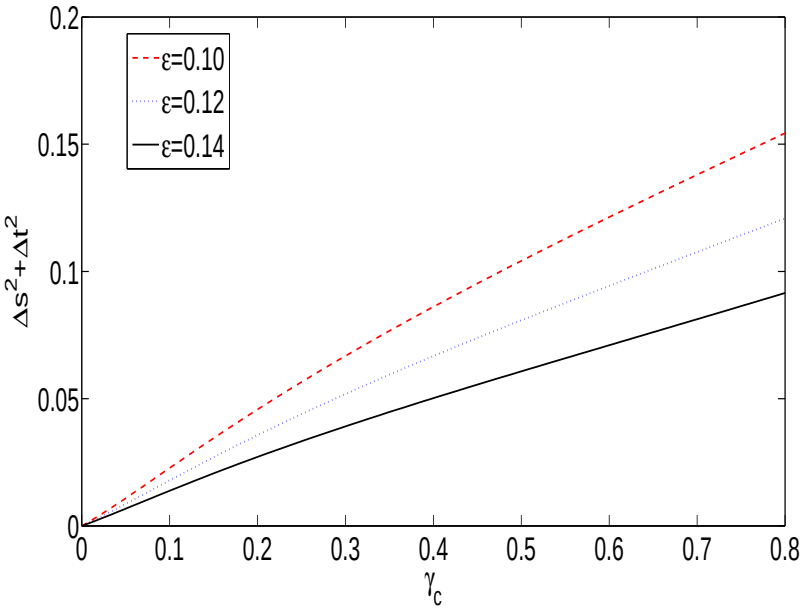

Figure 4: Plots of photon entanglement [Eq. (81)] versus γ_c for $\tau_a = 0.4,\ \kappa = 0.8,\ N = 50,\ \gamma = 0.2$, and for different values of ε.

On the other hand, cavity atoms of a system are entangled, if the sum of the variance of a pair of EPR-like operators,

$$\hat{u} = \hat{X}_2' - \hat{X}_1', \tag{83}$$

$$\hat{v} = \hat{P}_2' + \hat{P}_1', \tag{84}$$

where $\hat{X}_1' = \frac{1}{\sqrt{2}}(\hat{\Sigma}_1 + \hat{\Sigma}_1^\dagger)$, $\hat{X}_2' = \frac{1}{\sqrt{2}}(\hat{\Sigma}_2 + \hat{\Sigma}_2^\dagger)$, $\hat{P}_1' = \frac{i}{\sqrt{2}}(\hat{\Sigma}_1^\dagger - \hat{\Sigma}_1)$, $\hat{P}_2' = \frac{i}{\sqrt{2}}(\hat{\Sigma}_2^\dagger - \hat{\Sigma}_2)$ are quadrature operators for the cavity atoms, satisfy

$$(\Delta u)^2 + (\Delta v)^2 < 2N. \tag{85}$$

Since $\hat{\Sigma}_1$ and $\hat{\Sigma}_2$ are Gaussian atomic operators with zero mean so one can easily find

$$(\Delta u)^2 + (\Delta v)^2 = \left[\langle\hat{\Sigma}_1^\dagger\hat{\Sigma}_1\rangle + \langle\hat{\Sigma}_1\hat{\Sigma}_1^\dagger\rangle + \langle\hat{\Sigma}_2^\dagger\hat{\Sigma}_2\rangle + \langle\hat{\Sigma}_2\hat{\Sigma}_2^\dagger\rangle\right] - \left[\langle\hat{\Sigma}_1\hat{\Sigma}_2\rangle + \langle\hat{\Sigma}_2^\dagger\hat{\Sigma}_1^\dagger\rangle\right]. \tag{86}$$

It then follows that

$$(\Delta u)^2 + (\Delta v)^2 = 2N - \langle\hat{N}_1\rangle - \langle\hat{N}_2\rangle. \tag{87}$$

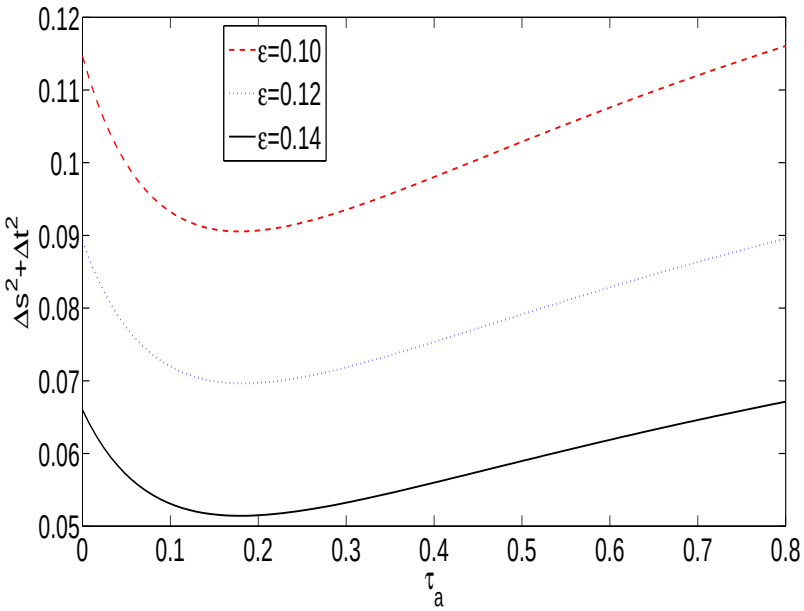

Figure 5: Plots of photon entanglement [Eq. (81)] versus τ_a for $\gamma_c = 0.4, \kappa = 0.8, N = 50, \gamma = 0.2$, and for different values of ε.

In the absence of subharmonic generator, Eq. (87) leads to

$$(\Delta u)^2 + (\Delta v)^2 = 2N - \left[\frac{(\gamma_c+\gamma)^2(\gamma+2\tau_a)+\gamma\tau_a^2}{(\gamma+\gamma_c)(\gamma+\tau_a)(\gamma+\gamma_c+\tau_a)}\right]N. \tag{88}$$

We immediately see note from Eqs. (82) and (88) that photon-state entanglement is greater than atom-state entanglement for the same values of κ, γ, γ_c, and τ_a.

VII. Conclusion

In this paper we present a thorough study of the squeezing as well as the statistical properties of the light produced by electrically pumped non-degenerate three-level laser, with two-mode subharmonic generator, coupled to a two-mode vacuum reservoirs via a single-port mirror whose open cavity contains N non-degenerate three-level cascade atoms. We carried out our analysis by putting the noise operators associated with a vacuum reservoir in normal order. We then first obtained the quantum Langevin equations for the cavity mode operators. We next determined the equations of evolution of the expectation values of atomic operators employing the pertinent master equation. Applying the steady-state solution of these equations, we have analyzed the mean photon number, the CV bipartite atomic and photon state entanglement as well as atom and photon number correlation. It is found that the photons and the atoms in the system are strongly entangled at steady state. Results show that the presence of parametric amplifier is to increase the squeezing and the mean photon number of the two-mode cavity light significantly. It is found that the photon- states of the system is strongly entangled at steady state where as the atomic state of the system is not entangled. In addition, we have established that the photons in the laser cavity are highly correlated and the degree of photon number correlation and entanglement increases as the amplitude of the coherent light driving the subharmonic generator increases. In addition, we have established that the presence of the subharmonic generator leads to an increase in the degree of entanglement and correlation. Moreover, we pointed out that in the absence of subharmonic generator, one can never realize correlated photons.

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Global Journal of Science Frontier Research: a
Physics and Space Science
Volume 18 Issue 9 Version 1.0 Year 2018
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Homopolar Induction in the Concept of the Scalar-Vector Potential

By F. F. Mende

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Keywords: *law of the induction of faraday, lorentz force, homopolar induction, unipolar generator, scalar- vector potential.*

GJSFR-A Classification: *FOR Code: 020399*

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I. Introduction

At present classical electrodynamics consists of two not connected together parts. This of Maxwell's equation the determining wave properties [1], pour on from one side, from other side these are Lorentz force [2], is determining ponderomotive (power) interactions of the current carrying systems. As is known, this force is introduced in the form separate postulate. And as yet there is no that connecting link, which would unite these odd parts. in the electrodynamics the fundamental law of induction is Farrday law. But also here matters be not in the best way. Homopolar induction is exception from this law, whiches indicate its incompleteness. Specifically, this induction, until now, causes among the scientists many disputes, it was opened by Faraday more than 200 years ago, but also up to now the physical principles of the operation of some constructions of unipolar generators remain obscure. There were the attempts to explain the work of such generators by action on the moving charges of Lorentz force, but it turned out that there are such constructions, in which to explain their operating principle thus is impossible. Here one example. If we to the conducting disk, fasten on the insulating plate end magnet, and to begin I utter its mating-call construction to revolve, then between the brushes, which slide through the periphery of disk and by its axis, will arise a potential difference. Since in the case indicated the magnet revolves together with the disk, there is no motion of the charges of disk in the magnetic field of magnet, and to explain the appearance of a voltage drop across brushes on the basis of Lorentz force is impossible.

In the separate publications it is indicated that for explaining the work of unipolar generators should be drawn the special theory of relativity (SR), but no one showed, as this to make. It will be in this work shown that attraction of the concept of scalar- vector potential for explaining the work of all known constructions of unipolar generators gives answers to all presented questions.

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II. Homopolar Induction In The Concept Of Scalar- Vector Potential

Beginning the study of the problem about the homopolar induction, it is necessary to clearly demarcate the concepts of a potential difference and electromotive force (EMF). The scalar potential of fixed charge is determined by the relationship

$$\varphi_0(r)=\frac{Q}{4\pi\varepsilon r},$$

where Q - magnitude of the charge, and ε - dielectric constant of medium.

Electric field is the gradient of the scalar potential

$$\mathbf{E}=-grad\ \varphi_0(r).$$

This field is potential, while this means that the work is not accomplished with the transfer of trial charge in this field along any locked trajectory, i.e. the condition is satisfied

The electromotive force is the scalar quantity, which characterizes the work of strange (nonpotential) forces in the locked conducting outline and is determined the work of these forces on the displacement of unit charge along the outline. EMF determines the work of these forces on the displacement of single positive charge along the outline. This work is determined by work EMF to the magnitude of the charge. Both potential difference and EMF are measured in volts.

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In the usual electric generators EMF is generated in the locked fixed or moving outline, partly which appears the load, in which is separated the energy. A difference in the unipolar generator from such generators is the fact that in it the locked outline is composite: one part of this outline is fixed, and the second moves relative to the first. Galvanic contact between these parts is ensured with the aid of the feeder brushes. Both parts of the outline of unipolar generator their EMF, which have different value, are excited. EMF in this composite outline is the bag EMF of its component parts. Let us note that component parts it can be and more than two.

The concept of scalar- vector potential, developed in the works [3-6], the dependence of the scalar potential of charge on its relative speed is assumed

$$\varphi(v)=\varphi_0 ch\frac{v_\perp}{c}, \qquad (1)$$

where $v_\perp$ - normal component of charge rate to the vector, which connects the moving charge and observation point, c - speed of light Let us examine the case, when there is a section of the conductor, along which flows the current (Fig.1). We will consider that in the conductor are two subsystems of the mutually inserted charges of positive lattice g^+ and free electrons g^-, absolute values of which they are equal in magnitude they compensate each other, leaving conductor neutral. For convenience in the examination in the figure these two subsystems are moved apart along the coordinate r.

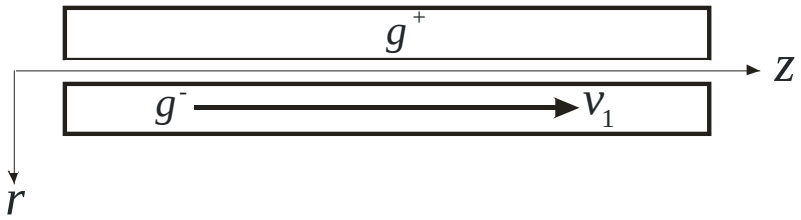

Fig. 1: Section is the conductor, along which flows the current

The electric field, created by rigid lattice depending on the distance of from the center of the conductor, that is located along the axis of it takes the form

$$E^+=\frac{g}{2\pi\varepsilon r}, \qquad (2)$$

where g - the absolute value of a quantity of positive chargex, which fall per unit of the length of conductor.

As in relationship (2), with the further consideration we will introduce only absolute values of the densities both of positive and negative charges, counting the absolute values of electrical pour on, which coincide in the direction from r by positive, and opposite to this direction - negative.

Using relationship (2), we obtain the values of electrical pour on, created by the electrons, which move in the conductor with the speed v_1

$$E^- = -\frac{g}{2\pi\varepsilon r} ch\frac{v_1}{c} \cong -\frac{g}{2\pi\varepsilon r}\left(1+\frac{1}{2}\frac{v_1^2}{c^2}\right). \qquad (3)$$

In this relationship only two first members of expansion in the series of hyperbolic cosine are undertaken.

Adding (2) and (3), we obtain the summary value of the electric field at a distance of from the axis of the conductor:

$$E=-\frac{gv_1^2}{4\pi\varepsilon c^2 r}.$$

This relationship indicates that around the conductor, along which move the electrons, is created the electric field, which corresponds to the negative charge of conductor. However, this field with those current densities, which can be provide ford in the normal conductors, has insignificant value, and discovered be it cannot with the aid of the existing measuring means. It can be discovered only with the use of the superconductors, where the current density can on many orders exceed currents in the normal metals [4].

Let us examine the case, when very section of the conductor, on which with the speed v_1 flow the electrons, moves in the opposite direction with speed v (Fig. 2).

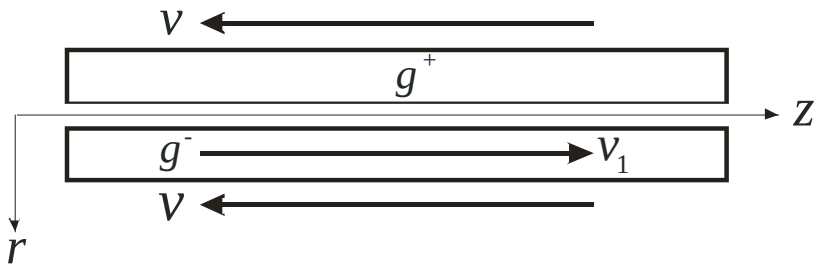

Fig. 2: Section of conductor with the current, which moves with the speed v

In this case relationships (2) and (3) will take the form

$$E^{+}=\frac{g}{2\pi\varepsilon r}\left(1+\frac{1}{2}\frac{v^2}{c^2}\right), \tag{4}$$

$$E^{-}=-\frac{g}{2\pi\varepsilon r}\left(1+\frac{1}{2}\frac{(v_1-v)^2}{c^2}\right). \tag{5}$$

Adding (4) and (5), we obtain the summary field

$$E_{\Sigma}=\frac{g}{2\pi\varepsilon r}\left(\frac{v_1 v}{c^2}-\frac{v_1^2}{c^2}\right). \tag{6}$$

We will consider that the speed of the mechanical motion of conductor is considerably more than the drift velocity of electrons. Then in relationship (6) the second term in the brackets can be disregarded, and finally we obtain:

$$E\cong\frac{gv_1 v}{2\pi\varepsilon c^2 r}. \tag{7}$$

The obtained result means that around the moving conductor, along which flows the current, with respect to the fixed observer is formed the electric field, determined by relationship (7), which is equivalent to appearance on this conductor of the specific positive charge of the equal This field not is potential, since it is created with strange sources, and it is equivalent to appearance on this conductor of the specific positive charge of the equal

$$g^{+}=\frac{gv_1 v}{c^2}. \tag{8}$$

If in parallel with the conductor with the same speed moves the plate (it is shown in the lower part of Fig. 3), whose width is equal r_2-r_1, then between its edges will be observed electromotive force

$$U_1=-\int_{r_1}^{r_2}\frac{gv_1^2 dr}{2\pi\varepsilon c^2 r}=-\frac{gv_1^2}{2\pi\varepsilon c^2}\ln\frac{r_2}{r_1}. \tag{9}$$

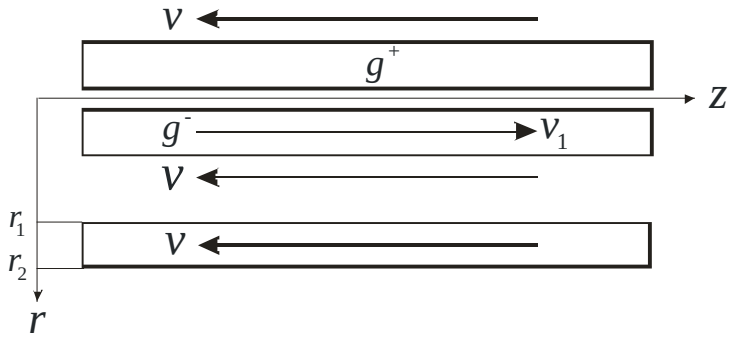

Fig. 3: The conducting plate moves with the same speed, as conductor

But if along the moving plate slide the contacts, which are fixed in the reference system (Fig.4), then EMF between such contacts will comprise

$$U_2=\frac{g}{2\pi\varepsilon}\left(\frac{v_1 v}{c^2}-\frac{1}{2}\frac{v_1^2}{c^2}\right)\ln\frac{r_2}{r_1}. \tag{10}$$

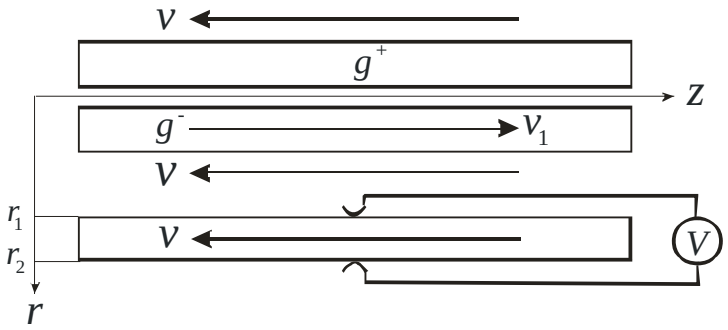

Fig. 4: To the conducting plate, which is moved together with the conductor, with the aid of the brushes the voltmeter is connected

The conducting plate, which is moved together with the conductor, presents together with the circuit of voltmeter the composite locked outline, in which will act EMF, which is been the sum of electromotive force, which is located on the component parts of the outline. We will obtain its value, summing up expressions (9) and (10):

$$U_{\Sigma}=U_2-U_1=\frac{gv_1v}{2\pi\varepsilon c^2}\ln\frac{r_1}{r_2}. \qquad (11)$$

If the conductor, along which flows the current, to roll up into the ring, after making from it a turn with the current, and to revolve this turn then, so that the speed of its parts would be equal v, that the electric field, which corresponds to the presence on the conductor of the specific charge, determined by the relationship will appear around this turn, (8). Let us roll up into the ring the conducting plate, after making from it a disk with the opening, and let us join to its generatrix feeder brushes, as shown in Fig. 5. If we with the identical speed revolve ring and disk, then on the condition that that the diameter of ring is considerably more than its width, on the brushes we will obtain EMF, determined by relationship (11).

Ring of wire can consist not of one, but of several turns. Such turns are the analog of the end magnet, which revolves together with the conducting disk.

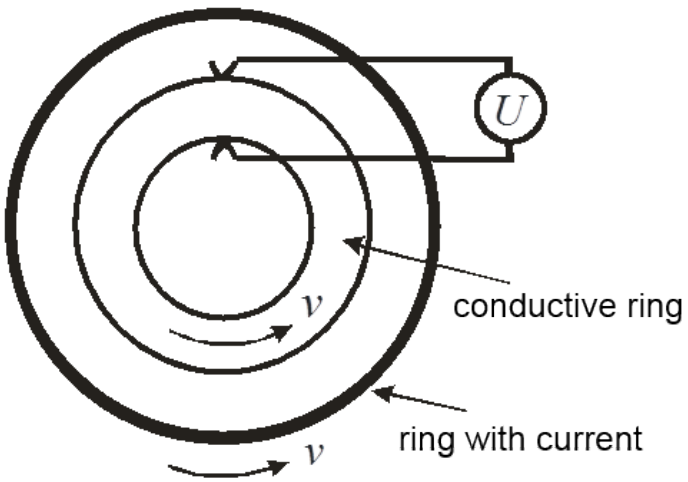

Fig. 5: Schematic of unipolar generator with the revolving turn with the current and the revolving conducting ring

Is examined the most contradictory version of the unipolar generator, the explanation of the operating principle of which in the literary sources previously was absent. With its examination it is not possible to use a concept of Lorentz force, since. and magnet and conducting ring revolve together with the identical speed.

The conducting ring and the revolving together with it magnet it is possible to combine in the united construction. For this should be taken the conducting disk from the magnetic material and magnetized it in the axial direction, after joining feeder brushes to its generatrix. The continuous magnetized disk is the limiting case of this construction. With this EMF it is removed with the aid of the feeder brushes between the generatrix of disk and its axis. This construction presents the unipolar generator, which was proposed still by Faraday.

III. Conclusion

The homopolar induction was discovered by Faraday more than 200 years ago, but also up to now the physical principles of the operation of some constructions of unipolar generators remain obscure. There were the attempts to explain the work of such generators by action on the moving charges of Lorentz force, but it turned out that there are such constructions, in which to explain their operating principle thus is impossible. It is in the present work shown that the concept of the scalar- vector potential, which assumes the dependence of the scalar potential of charge on its relative speed, makes it possible to explain the work of all existing types of unipolar generators. from the day of discovery by Faradayhomopolar induction this article appears first and thus far of only, where the true reasons for this phenomenon are reflected.

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Global Journal of Science Frontier Research: a
Physics and Space Science
Volume 18 Issue 9 Version 1.0 Year 2018
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

A Classification of Quantum Particles

By Vu B Ho

Abstract- In this work, by summarising our recent works on the differential geometric and topological structures of quantum particles and spacetime manifolds, we discuss the possibility to classify quantum particles according to their intrinsic geometric structures associated with differentiable manifolds that are solutions to wave equations of two and three dimensions. We show that fermions of half-integer spin can be identified with differentiable manifolds which are solutions to a general two-dimensional wave equation, in particular, a two dimensional wave equation that can be derived from Dirac equation. On the other hand, bosons of integer spin can be identified with differentiable manifolds which are solutions to a general three-dimensional wave equation, in particular, a three-dimensional wave equation that can be derived from Maxwell field equations of electromagnetism. We also discuss the possibility that being restricted to three-dimensional spatial dimensions we may not be able to observe the whole geometric structure of a quantum particle but rather only the cross-section of the manifold that represents the quantum particle and the space in which we are confined. Even though not in the same context, such view of physical existence may comply with the Copenhagen interpretation of quantum mechanics which states that the properties of a physical system are not definite but can only be determined by observations.

GJSFR-A Classification: *FOR Code: 020699*

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I. Covariant Formulations Of Classical And Quantum Physics

In physics, the electromagnetic field has a dual character and plays a crucial role both in the formulation of relativity theory and quantum mechanics. However, since the electromagnetic field itself is regarded simply as a physical event whose dynamics can be described by mathematical methods therefore it is reasonable to suggest that it should be formulated in both forms of classical and quantum mathematical formulations. This amounts to suggesting that it should be derived from the same mathematical structure of classical theories, such as the gravitational field, and at the same time from the same mathematical structure of quantum theories, such as Dirac formulation of quantum mechanics. In this section we show that in fact this is the case. As shown in our works on spacetime structures of quantum particles [1], the three main dynamical descriptions of physical events in classical physics, namely Newton mechanics, Maxwell electromagnetism and Einstein gravitation, can be formulated in the same general covariant form and they can be represented by the general equation

$$\nabla_\beta M = kJ \tag{1}$$

where M is a mathematical object that represents the corresponding physical system and ∇_β is a covariant derivative. For Newton mechanics, we have $M = E = \frac{1}{2}m\sum_{\mu=1}^{3}(dx^\mu/dt)^2 + V$ and $J = 0$. For Maxwell electromagnetism, $M = F^{\alpha\beta} = \partial^\mu A^\nu - \partial^\nu A^\mu$ with the four-vector potential $A^\mu \equiv (V, \mathbf{A})$ and J can be identified with the electric and magnetic currents. And for Einstein gravitation, $M = R^{\alpha\beta}$ and J can be defined in terms of a metric $g_{\alpha\beta}$ and the Ricci scalar curvature. It is shown in differential geometry that the Ricci tensor $R^{\alpha\beta}$ satisfies the Bianchi identities

$$\nabla_\beta R^{\alpha\beta} = \frac{1}{2}g^{\alpha\beta}\nabla_\beta R \tag{2}$$

where $R = g^{\alpha\beta}R_{\alpha\beta}$ is the Ricci scalar curvature [2]. Even though Equation (2) is purely geometrical, it has a covariant form similar to the electromagnetic tensor $\partial_\alpha F^{\alpha\beta} = \mu j^\beta$ defined in Euclidean space. If the quantity $\frac{1}{2}g^{\alpha\beta}\nabla_\beta R$ can be identified as a physical entity, such as a four-current of gravitational matter, then Equation (2) has the status of a dynamical law of a physical theory. In this case a four-current $j^\alpha = (\rho, \mathbf{j}_i)$ can be defined purely geometrical as

$$j^\alpha = \frac{1}{2}g^{\alpha\beta}\nabla_\beta R \tag{3}$$

If we use the Bianchi identities as field equations for the gravitational field then Einstein field equations, as in the case of the electromagnetic field, can be regarded as a definition for the energy-momentum tensor $T_{\mu\nu}$ for the gravitational field [3]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\alpha\beta} = \kappa T_{\mu\nu} \tag{4}$$

For a purely gravitational field in which $\frac{1}{2}g^{\alpha\beta}\nabla_\beta R = 0$, the proposed field equations given in Equation (2) also give rise to the same results as those obtained from Einstein formulation of the gravitational field given in Equation (4). For a purely gravitational field, Equation (2) reduces to the equation

$$\nabla_\beta R^{\alpha\beta} = 0 \tag{5}$$

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From Equation (5), we can obtain solutions found from the original Einstein field equations, such as Schwarzschild solution, by observing that since $\nabla_\mu g^{\alpha\beta} \equiv 0$, Equation (5) implies

$$R_{\alpha\beta} = \Lambda g_{\alpha\beta} \tag{6}$$

where Λ is an undetermined constant. Furthermore, the intrinsic geometric Ricci flow that was introduced by Hamilton can also be derived from Equation (5) and given as follows

$$\frac{\partial g_{\alpha\beta}}{\partial t} = \kappa R_{\alpha\beta} \tag{7}$$

where κ is a scaling factor. Mathematically, the Ricci flow is a geometric process that can be employed to smooth out irregularities of a Riemannian manifold [4]. There is an interesting feature that can be derived from the definition of the four-current $j^\alpha=(\rho, \mathbf{j}_i)$ given in Equation (3). By comparing Equation (3) with the Poisson equation for a potential V in classical physics $\nabla^2 V=4\pi\rho$, we can identify the scalar potential V with the Ricci scalar curvature R and then obtain a diffusion equation

$$\frac{\partial R}{\partial t} = k\nabla^2 R \tag{8}$$

where k is an undetermined dimensional constant. Solutions to Equation (8) can be found to take the form [5]

$$R(x,y,z,t) = \frac{M}{\left(\sqrt{4\pi kt}\right)^3} e^{-\frac{x^2+y^2+z^2}{4kt}} \tag{9}$$

Equation (9) determines the probabilistic distribution of an amount of geometrical substance M which is defined via the Ricci scalar curvature R and manifests as observable matter. It is interesting to note that in fact it is shown that a similar diffusion equation to Equation (8) can also be derived from the Ricci flow given in Equation (7) as follows [6]

$$\frac{\partial R}{\partial t} = \Delta R + 2|\mathrm{Ric}|^2 \tag{10}$$

where Δ is the Laplacian defined as $\Delta=g^{\alpha\beta}\nabla_\alpha\nabla_\beta$ and $|\mathrm{Ric}|$ is a shorthand for a mathematical expression. Therefore, the Bianchi field equations of general relativity in the covariant form given in Equation (2) can be used to formulate quantum particles as differentiable manifolds, in particular 3D differentiable manifolds.

On the other hand, we have also shown that Maxwell field equations of electromagnetism and Dirac relativistic equation of quantum mechanics can be formulated covariantly from a general system of linear first order partial differential equations [7,8,9]. An explicit form of a system of linear first order partial differential equations can be written as follows [10]

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}^r \frac{\partial \psi_i}{\partial x_j} = k_1 \sum_{l=1}^{n} b_l^r \psi_l + k_2 c^r, \qquad r = 1, 2, \ldots, n \tag{11}$$

The system of equations given in Equation (11) can be rewritten in a matrix form as

$$\left(\sum_{i=1}^{n} A_i \frac{\partial}{\partial x_i}\right)\psi = k_1\sigma\psi + k_2 J \tag{12}$$

where $\psi = (\psi_1, \psi_2, \ldots, \psi_n)^T$, $\partial\psi/\partial x_i = (\partial\psi_1/\partial x_i, \partial\psi_2/\partial x_i, \ldots, \partial\psi_n/\partial x_i)^T$, A_i, σ and J are matrices representing the quantities a_{ij}^k, b_l^r and c^r and k_1 and k_2 are undetermined constants. Now, if we apply the operator $\sum_{i=1}^n A_i \frac{\partial}{\partial x_i}$ on the left on both sides of Equation (12) then we obtain

$$\left(\sum_{i=1}^{n} A_i \frac{\partial}{\partial x_i}\right)\left(\sum_{j=1}^{n} A_j \frac{\partial}{\partial x_j}\right)\psi = \left(\sum_{i=1}^{n} A_i \frac{\partial}{\partial x_i}\right)(k_1\sigma\psi + k_2 J) \tag{13}$$

If we assume further that the coefficients a_{ij}^k and b_l^r are constants and $A_i\sigma = \sigma A_i$, then Equation (13) can be rewritten in the following form

$$\left(\sum_{i=1}^{n} {A_i}^2 \frac{\partial^2}{\partial x_i^2} + \sum_{i=1}^{n}\sum_{j>i}^{n} (A_iA_j + A_jA_i)\frac{\partial^2}{\partial x_i \partial x_j}\right)\psi = k_1^2\sigma^2\psi + k_1k_2\sigma J + k_2\sum_{i=1}^{n} A_i \frac{\partial J}{\partial x_i} \tag{14}$$

In order for the above systems of partial differential equations to be used to describe physical phenomena, the matrices A_i must be determined. We have shown that for both Dirac and Maxwell field equations, the matrices A_i must take a form so that Equation (14) reduces to the following equation

$$\left(\sum_{i=1}^{n} {A_i}^2 \frac{\partial^2}{\partial x_i^2}\right)\psi = k_1^2\sigma^2\psi + k_1k_2\sigma J + k_2\sum_{i=1}^{n} A_i\frac{\partial J}{\partial x_i} \tag{15}$$

Therefore, the general system of linear first order partial differential equations can be used to formulate the dynamics of quantum fields that include the electromagnetic field and matter field of quantum particles. In Sections 3 and 5 we will give explicit forms for the matrices A_i for both Dirac and Maxwell field equations and show that these two systems of differential equations can be applied to classify quantum particles as fermions of half-integer spin and bosons of integer spin.

II. On The Dimensionality Of The Spatiotemporal Manifold

In classical physics, in order to formulate the dynamics of natural events that are observable we assume that spacetime is a continuum which consists of three spatial dimensions and one temporal dimension. At the macroscopic scale on which information about physical objects can be established with certainty the assumption seems to be reasonable because it can adequately be used for all dynamical formulations of physical theories. However, at the microscopic scale quantum responses of physical events have revealed that such simple picture of a four-dimensional spacetime continuum is in fact not adequate for physical descriptions, especially those that can be accounted for by observations that can only be set up within our perception of physical existence. This leads to a more fundamental problem in physical investigation of how we can justify the merit of a physical theory. From the perspective of scientific investigation, physical theories can only be evaluated on the subject of the accuracy to experimental results of their mathematical formulations that can be applied into the dynamical description of physical objects. But as far as we are concerned, the setup of a physical experiment is within the limit of three-dimensional domain, therefore, the dimensionality of the spatiotemporal manifold in fact still remains the most fundamental problem that needs to be addressed before any attempt to formulate physical theory can be justified. In our previous works on spacetime structures of quantum particles and geometric interactions we showed that it is possible to formulate quantum particles as three-dimensional differentiable manifolds which have further geometric and topological structure of a CW complex whose decomposed *n*-cells can be associated with physical fields that form the fundamental physical interactions between physical objects [11,12 ,13]. We also showed that it is possible to suggest that spacetime as a whole is a fiber bundle which admits different types of fibers for the same base space of spacetime and what we are able to observe are the dynamics of the fibers but not that of the base space itself [14]. Even though the fiber bundle formulation of the spatiotemporal manifold may provide a more feasible framework to deal with the dynamics of physical existence, the questions about the nature of the base space of the spatiotemporal fiber bundle, whether it can be observable and whether matter are physical entities or they are simply geometric and topological structures of the spacetime manifold still remain unanswered. We may also ask the question of how many dimensions the universe really has then even though the answer to this type of question will depend on our epistemological approach to the physical existence, within our geometric and topological formulation of spacetime we would say that it would depend on what is the highest dimension of the *n*-cells that are decomposed from the spacetime bundle that we can perceive. However, it seems natural that being apparently three-dimensional we perceive the physical existence in three spatial dimensions. It is also natural that due to our perception of the progress of physical events that occur in sequence that we recognise time as one-dimensional. In physics, practically, we describe the physical existence in terms of those that can be observed and measured. In classical physics, what we are observing are physical objects that move in a three-dimensional Euclidean space and the motion occurs in sequence that changes spatial position with respect to time, which itself can be measured by using the displacement of the physical objects. However, in quantum physics, the observation of physical objects itself is a new epistemological problem. The fundamental issue that is related to this epistemological problem of observation is the difficulty in knowing how quantum particles exist. In Einstein theory of special relativity the dimensionality of spacetime is assumed to be that of one-dimensional time and three-dimensional Euclidean space R^3, together they form a four-dimensional spacetime which has the Minkowski mathematical structure of pseudo-Euclidean geometry. This mathematical structure seems to be complete in itself if spacetime is not curved. However, in Einstein theory of general relativity, spacetime is assumed to be curved by matter and energy. As a consequence from the assumption of the Minkowski mathematical structure

of a four-dimensional spacetime, the mathematical objects that are used to describe physical objects can only be described as two-dimensional manifolds embedded in the three-dimensional Euclidean space R^3. In fact, as shown in Section 3 below, this may be true for the case of massive quantum particles of half-integer spin. On the other hand, in order to describe physical objects that are assumed to possess the mathematical structures of three-dimensional manifolds an extra dimension of space must be used. For example, with Einstein field equations given as $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \quad g_{\mu\nu} = \kappa T_{\mu\nu}$ and the cosmological model that uses the Robertson-Walker metric of the form $ds^2 = c^2dt^2 - S^2(t)\left(dr^2/(1-kr^2) + r^2\right.$, $\left.(d\theta^2 + \sin^2\theta d\phi^2)\right)$, in order to derive the Robertson-Walker metric from Einstein field equations we assume that the quantity $S(t)$ is the radius of a 3-sphere embedded in a four-dimensional Euclidean space R^4. This raises the question of whether this extra spatial dimension is real or just for convenience. Furthermore, we may ask whether there are any other physical formulations of physics that also require an extra dimension of space. This is in fact also the case when we discussed the wave-particle duality in quantum mechanics in which quantum particles can be assumed to possess the geometric and topological structures of a three-dimensional differentiable manifold [15]. As a matter of fact, the anthropic cognition of spacetime with higher dimensions is a subject of scientific investigation and with open-mindedness there is no reason why we should avoid any attempt to formulate a physical theory that requires such perception with reasoning thinking. Even though the CW complex and fiber bundle formulations give a general description of the geometric structures of both quantum particles and the spatiotemporal manifold, the more important question that still remains is how to determine the specific structure of each quantum particle. For example, if quantum particles are considered as three-dimensional differentiable manifolds then it is reasonable to suggest that generally their geometric structures should be classified according to Thurston geometries [16]. However, even with a correct classification of quantum particles according to their intrinsic geometric structures, this type of geometric classification lacks the more important aspects of physical descriptions that are required for a physical theory which encompasses the dynamics and the interactions between them. With the assumption that quantum particles possess the intrinsic geometric structures of a CW complex and each geometric structure manifests a particular type of physical interactions, it is reasonable to assume that there is a close relationship between geometric structures in terms of decomposed *n*-cells from a CW complex and physical interactions. In general, we may consider physical objects of any scale as differentiable manifolds of dimension n which can emit submanifolds of dimension $m \leq n$ by decomposition. In order to formulate a physical theory we would need to devise a mathematical framework that allows us to account for the amount of subspaces that are emitted or absorbed by a differentiable manifold. This is the evolution of a geometric process that manifests as a physical interaction. We assume that an assembly of cells of a specified dimension will give rise to a certain form of physical interactions and the intermediate particles, which are the force carriers of physical fields decomposed during a geometric evolution, may possess a specified geometric structure, such as that of the *n*-spheres and the n-tori. Therefore, for observable physical phenomena, the study of physical dynamics reduces to the study of the geometric evolution of differentiable manifolds. In particular, if a physical object is considered to be a three-dimensional manifold then there are four different types of physical interactions that are resulted from the decomposition of 0-cells, 1-cells, 2-cells and 3-cells and these cells can be associated with the corresponding spatial forces $F_n = k_n r^n$ and temporal forces $F_n = h_n t^n$ with $-3 \leq n \leq 3$. In the case of $n = 0$, for a definite perception of a physical existence, we assume that space is occupied by mass points which interact with each other through the decomposition of 0-cells. However, since 0-cells have dimension zero therefore there are only contact forces between the mass points. When the mass points join together through the contact forces they form elementary particles. The 0-cells with contact forces can be arranged to form a particular topological structure [17]. Therefore, we can assume that a general spatiotemporal force which is a combination of the spatial and temporal forces resulted from the decomposition of spatio-temporal *n*-cells of all dimensions can take the form

$$F = \sum_{n=-3}^{3} (k_n r^n + h_n t^n) \tag{16}$$

where k_n and h_n are constants which can be determined from physical considerations. Using equations of motion from both the spatial and temporal Newton's second laws of motion

$$m\frac{d^2\mathbf{r}}{d\tau^2} = \mathbf{F} \tag{17}$$

$$D\frac{d^2\mathbf{t}}{ds^2} = \mathbf{F} \tag{18}$$

it is seen that a complete geometric structure would be the structure that is resulted from the relationship between space and time that satisfies the most general equation in the form

$$m\frac{d^2\mathbf{r}}{d\tau^2}+D\frac{d^2\mathbf{t}}{ds^2}=\sum_{n=-3}^{3}(k_n r^{n-1}\mathbf{r}+h_n t^{n-1}\mathbf{t}) \tag{19}$$

The above discussions suggest that the apparent geometric and topological structures of the total spatiotemporal manifold are due to the dynamics and the geometric interactions of the decomposed cells from the base space of the total spatiotemporal manifold, and the decomposed cells form different types of fibers which may also geometrically interact with each other and manifest as physical interactions. In this case we can only perceive the appearance of the intrinsic geometric structures that emerge on the base space of the total spatiotemporal manifold and the base space itself may not be observable with the reasonable assumption that a physical object is not observable if it does not have any form of geometric interactions. It could be that the base space of the spatiotemporal manifold at the beginning was only a six-dimensional Euclidean spatiotemporal continuum R^6 which had no non-trivial geometric structures therefore contained no physical objects. How could physical objects be formed from such a plain spacetime continuum? Even though we could suggest that physical objects could be formed as three-dimensional differentiable manifolds from mass points with contact forces associated with the decomposed 0-cells, it is hard to imagine how they can be formed from a plain continuum without assuming that there must be some form of spontaneous symmetry breaking of the vacuum. Since the apparent spacetime structures are formed by decomposed cells from the base spacetime and since there are many different relationships that arise from the geometric interactions of the decomposed cells of different dimensions, therefore there are different spactime structures each of which can represent a particular spacetime structure and all apparent spacetime structures can be viewed as parallel universes of a multiverse. If we assume that the spatiotemporal manifold is described by a six-dimensional differentiable manifold which is composed of a three-dimensional spatial manifold and a three-dimensional temporal manifold, in which all physical objects are embedded, then the manifold M can be decomposed in the form $M=M\#S_S^3\#S_T^3$, where S_S^3 and S_T^3 are spatial and temporal 3-spheres, respectively. Despite this form of decomposition can be used to describe gravity as a global structure it cannot be used as a medium for any other physical fields which possess a wave character. Therefore we would need to devise different types of decomposition to account for these physical fields that require a local geometric structure. For example, we may assume that *n*-cells can be decomposed from the spatiotemporal manifold at each point of the spatiotemporal continuum. This is equivalent to considering the spatiotemporal manifold as a fiber bundle $E=B\times F$, where B is the base space, which is the spatiotemporal continuum, and the fiber F, which is the *n*-cells. We will discuss in more details in Section 4 the local geometric and topological structure of the spatiotemporal manifold when we discuss the possibility to formulate a medium for the electromagnetic field in terms of geometric structures. From the above discussions on the dimensionality of spacetime it is clear that the observation of natural events needs to be addressed. It seems that due to our physical existence we do not have the ability to observe a complete picture of a physical object. We can only observe part of a physical object due to the fact that it may exist in a higher spatial dimension than ours. For example, if quantum particles exist as three-dimensional differentiable manifold embedded into a four-dimensional Euclidean space then we are unable to observe the physical object as a whole but only the cross-section of it. We can use mathematics to determine the whole structure of the object but we cannot measure what we can calculate. The seemingly strange behaviour of quantum particles may also be caused by bringing over their classical model into the quantum domain. For example, when interacting with a magnetic field an elementary particle shows that it has some form of dynamics that can only be represented by intrinsic angular momentum that is different from the angular momentum encountered in classical physics in which elementary particles are assumed to be simply mass points without any internal geometric structure. In the next section we will show that half-integer values of the intrinsic angular momentum of an elementary particle can be obtained by taking into account its possible internal geometric and topological structures.

III. Quantum Particles With Half Integer Spin

In this section we will discuss a possible physical structure possessed by a quantum particle of half-integer spin that exists in three-dimensional space. If quantum particles are considered as differentiable manifolds then they should have intrinsic geometric structures, therefore, in terms of physical formulations they are composite physical objects. As suggested in Section 2 on the geometric interactions, a composite physical object can be formed from mass points by contact forces associated with the 0-cells decomposed from the CW complex that represents the quantum particle. The intrinsic geometric structure can be subjected to a geometric evolutionary process which manifests as the dynamics of the mass points that form the quantum particle. The manifested physical process may be described as that of a fluid dynamics that can be formulated in terms of a potential, like the Coulomb

potential of the electrostatic interaction in classical electrodynamics. Also discussed in Section 3 on the dimensionality of spacetime and the observability of quantum particles, physical objects can be observed completely if they can be described by a two-dimensional wave equation in which the solutions of the wave equation gives the description of the geometric structures of the physical object in a third spatial dimension. We now show how they can be obtained from a general two-dimensional wave equation, from two-dimensional Schrödinger wave equation and from Dirac equation in relativistic quantum mechanics. In particular, we will show that the two - dimensional Schrödinger wave equation does describe quantum particles with half-integer spin. Consider a quantum particle whose mass distribution is mainly on a two-dimensional membrane and whose charge is related to the vibration of a homotopy class of 2-spheres in which the charge can be described topologically in terms of surface density. The circular membrane is assumed to be made up of mass points that join together by contact forces which allow vibration. Without vibrating the membrane is a perfect two dimensional physical object, however when it vibrates it becomes a three dimensional physical object described as a two-dimensional manifold embedded in three-dimensional Euclidean space R^3. In this section we discuss the geometric structure of the quantum particle with regard to its distribution of mass and in the next section we will discuss the topological structure with regard to its distribution of charge density in terms of the homotopy fundamental group of surfaces. In this section we assume that a spacetime has three spatial dimensions and one temporal dimension. In general, the wave dynamics of a physical system in a two-dimensional space can be described by a wave equation written in the Cartesian coordinates (x, y) as

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial y^2} = 0 \tag{20}$$

In particular, Equation (20) can be used to describe the dynamics of a vibrating membrane in the (x, y) - plane. If the membrane is a circular membrane of radius then the domain D is given as $D=\{x^2+y^2<a^2\}$. In the polar coordinates given in terms of the Cartesian coordinates (x, y) as $x = r\cos\theta$, $y = r\sin\theta$, the two-dimensi- as

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\psi}{\partial r^2} - \frac{1}{r}\frac{\partial\psi}{\partial r} - \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} = 0 \tag{21}$$

The general solution to Equation (21) for the vibrating circular membrane with the condition $\psi=0$ on the boundary of can be found as [5]

$$\begin{aligned}\psi(r,\theta,t) = \sum_{m=1} J_0\left(\sqrt{\lambda_{0m}}r\right)\left(C_{0m}\cos\sqrt{\lambda_{0m}}ct + D_{0m}\sin\sqrt{\lambda_{0m}}ct\right)\\ + \sum_{m,n=1} J_n\left(\sqrt{\lambda_{nm}}r\right)(A_{nm}\cos n\theta\\ + B_{nm}\sin n\theta)\left(\left(C_{nm}\cos\sqrt{\lambda_{nm}}ct + D_{nm}\sin\sqrt{\lambda_{nm}}ct\right)\right)\end{aligned} \tag{22}$$

where $J_n\left(\sqrt{\lambda_{nm}}r\right)$ is the Bessel function of order n and the quantities A_{nm}, B_{nm}, C_{nm} and D_{nm} can be specified by the initial and boundary conditions. It is also observed that at each moment of time the vibrating membrane appears as a 2D differentiable manifold which is a geometric object whose geometric structure can be constructed using the wavefunction given in Equation (22). We now show that the curvature of the surfaces obtained from the vibrating membrane at each moment of time can also be expressed in terms of the derivatives of the wavefunction given in Equation (22). In differential geometry, the Ricci scalar curvature is shown to be related to the Gaussian curvature K by the relation $R=2K$, where K is expressed in terms of the principal radii k_1 and k_2 of a surface as $K = 1/k_1k_2$. Consider a surface defined by the relation $z=\psi(x,y)$ in Cartesian coordinates (x, y, z). The Ricci scalar curvature R can be found as [18]

$$R = \frac{2(\psi_{11}\psi_{22} - (\psi_{12})^2}{(1+\psi_1^2+\psi_2^2)^2} \tag{23}$$

where $\psi_\mu=\partial\psi/\partial x^\mu$ and $\psi_{\mu\nu} = \partial^2\psi/\partial x^\mu\partial x^\nu$. It is seen that the wavefunction $\psi(r,\theta,t)$ that is obtained from the wave equation given in Equation (22) can be used to determine the Ricci scalar curvature of a surface, which shows that the geometric structure of the vibrating membrane can be described by a classical wavefunction. In other words, wavefunctions that describe the wave motion of a vibrating membrane can be considered as a representation of physical objects. For the benefit of representation in the next section we now give a brief discussion on the geometric formation of quantum particles from a wave equation. We assumed that the circular membrane is made up of particles which are connected with each other by an

elastic force. This assumption leads to a more general hypothesis that a vibrating object is made up of mass points that join together by contact forces. When the membrane vibrates it takes different shapes at each moment of time. Each shape is a 2D differentiable manifold that is embedded in the three-dimensional Euclidean space. Now, if we consider the whole vibrating membrane as a particle then its geometric structure is described by the wavefunction ψ. It is a time-dependent hypersurface embedded in a three-dimensional Euclidean space. Now imagine an observer who is a two-dimensional object living in the plane (x, y) and who wants to investigate the geometric structure of the vibrating membrane. Even though he or she would not be able to observe the shapes of the embedded 2D differentiable manifolds in the three-dimensional Euclidean space, he or she would still be able to calculate the value of the wavefunction ψ at each point (x, y) that belongs to the domain $x^2+y^2<a^2$. What would the observer think of the nature of the wavefunction ψ? Does it represent a mathematical object, such as a third dimension, or a physical one, such as fluid pressure? Firstly, because the wavefunction ψ is a solution of a wave equation therefore it must be a wave. Secondly, if the observer who is a 2D physical object and who does not believe in higher dimensions then he or she would conclude that the wavefunction ψ should only be used to describe events of physical existence other than space and time. In the next section we will show that this situation may in fact be that of the wave-particle duality that we are encountering in quantum physics when our view of the physical existence is restricted to that of a 3D observer. It is also observed that according to the 2D observer who is living on the (x, y)-plane, the vibrating membrane appears as an oscillating motion of a single string. If the vibrating string is set in motion in space then it can be seen as a particle. With a suitable experimental setup, the moving vibrating membrane may be detected as a wave. And furthermore, it can also generate a physical wave if the space is a medium. In fact, as shown in the following, a two-dimensional wave equation can be applied into quantum mechanics to describe the dynamics of a quantum system which is restricted to a two-dimensional space. This can be formulated either by the Schrödinger non-relativistic wave equation or Dirac relativistic wave equation. However, in order to obtain a classical picture of a quantum particle in two-dimensional space, let consider the classical dynamics of a particle moving in two spatial dimensions. In classical mechanics, expressed in plane polar coordinates, the Lagrangian of a particle of mass m under the influence of a conservative force with potential $V(r)$ is given as follows [19]

$$L = \frac{1}{2}m\left(\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2\right) - V(r) \qquad (24)$$

With the Lagrangian given in Equation (24), the canonical momentum p_θ is found as

$$p_\theta = \frac{\partial L}{\partial(d\theta/dt)} = mr^2\frac{d\theta}{dt} \qquad (25)$$

The canonical momentum given in Equation (25) is the angular momentum of the system. By applying the Lagrange equation of motion

$$\frac{d}{dt}\frac{\partial L}{\partial(dq_i/dt)} - \frac{\partial L}{\partial q_i} = 0, \ \ i = 1, 2, \ldots, n \qquad (26)$$

where q_i are the generalised coordinates, we obtain

$$\frac{dp_\theta}{dt} = \frac{d}{dt}\left(mr^2\frac{d\theta}{dt}\right) = 0 \qquad (27)$$

The areal velocity $d\mathbf{A}/dt$, which is the area swept out by the position vector of the particle per unit time, is found as

$$\frac{d\mathbf{A}}{dt} = \frac{\mathbf{r}(t)\times\mathbf{v}(t)}{2} \qquad (28)$$

On the other hand, in classical dynamics, the angular momentum of the particle is defined by the relation

$$\mathbf{L} = \mathbf{r}(t)\times m\mathbf{v}(t) \qquad (29)$$

From Equations (28) and (29), we obtain the following relationship between the angular momentum $\mathbf{L}$ of a particle and the areal velocity $d\mathbf{A}/dt$

$$\mathbf{L} = 2m\frac{d\mathbf{A}}{dt} \qquad (30)$$

It is seen from these results that the use of conservation of angular momentum for the description of the dynamics of a particle can be replaced by the conservation of areal velocity. For example, consider the circular motion of a particle under an inverse square field $F = kq^2/r^2$. Applying Newton's second law, we obtain

$$\frac{mv^2}{r} = \frac{kq^2}{r^2} \qquad (31)$$

Using Equations (30) and (31) and the relation $L = mrv$, we obtain

$$r = \frac{4m}{kq^2}\left(\frac{dA}{dt}\right)^2 \qquad (32)$$

The total energy E of the particle is

$$E = \frac{1}{2}mv^2 - \frac{kq^2}{r} = -\frac{kq^2}{2r} \quad (33)$$

Using Equations (32) and (33), the total energy can be rewritten as

$$E = -\frac{k^2q^4}{8m\left(\frac{dA}{dt}\right)^2} \quad (34)$$

It is seen from Equation (34) that the total energy of the particle depends on the rate of change of the area dA/dt. In the case of Bohr model of a hydrogen-like atom, from the quantisation condition $mrv = n/2\pi$, we have

$$\frac{dA}{dt} = n\left(\frac{h}{4\pi m}\right) \quad (35)$$

Equation (35) shows that the rate of change of the area swept out by the electron is quantised in unit of $h/4\pi m$. The two-dimensional Bohr model of a hydrogen atom has a classical configuration that provides a clear picture of the motion of the electron around a nucleus. As shown in our work on the quantization of angular momentum, the Schrödinger wave mechanics when applied to the two-dimensional model of the hydrogen atom also predicts that an intrinsic angular momentum of the electron must take half-integral values for the Bohr spectrum of energy to be retained [20]. Using the two-dimensional model of the hydrogen atom, in the following we will describe an elementary particle of half-integer spin as a differentiable manifold whose physical configuration is similar to that of a rotating membrane whose dynamics can be described in terms of the two-dimensional motion using the Schrödinger wave mechanics and Dirac relativistic quantum mechanics. First, if elementary particles are assumed to possess an internal structure that has the topological structure of a rotating membrane then it is possible to apply the Schrödinger wave equation to show that they can have spin of half-integral values. Consider an elementary particle whose physical arrangement can be viewed as a planar system whose configuration space is multiply connected. Since the system is invariant under rotations therefore we can invoke the Schrödinger wave equation for an analysis of the dynamics of a rotating membrane. In wave mechanics the time-independent Schrödinger wave equation is given as [21]

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi(r) - V(r)\psi(r) = E\psi(r) \quad (36)$$

If we also assume that the overall potential $V(r)$ that holds the membrane together has the form $V(r) = A/r$, where A is a physical constant that is needed to be determined, then using the planar polar coordinates in two-dimensional space, the Schrödinger wave equation takes the form [22]

$$-\frac{\hbar^2}{2\mu}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}\right]\psi(r,\phi) - \frac{A}{r}\psi(r,\phi) = E\psi(r,\phi) \quad (37)$$

Solutions of the form $R(r)\Phi(\phi) = R(r)\Phi(\phi)$ reduce Equation (37) to two separate equations for the functions $\Phi(\phi)$ and $R(r)$ as follows

$$\frac{d^2\Phi}{d\phi^2} + m^2\Phi = 0 \quad (38)$$

$$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} - \frac{m^2}{r^2}R + \frac{2\mu}{\hbar^2}\left(\frac{A}{r} + E\right)R = 0 \quad (39)$$

where m is identified as the intrinsic angular momentum of the membrane. Equation (38) has solutions of the form

$$\Phi(\phi) = \exp(im\phi) \quad (40)$$

Normally, the intrinsic angular momentum must take integral values for the single-valuedness condition to be satisfied. However, if we consider the configuration space of the membrane to be multiply connected and the polar coordinates have singularity at the origin then the use of multivalued wavefunctions is allowable. As shown below, in this case, the intrinsic angular momentum m can take half-integral values. If we define, for the case $E < 0$,

$$\rho = \left(\frac{8\mu(-E)}{\hbar^2}\right)^{1/2} r, \quad \lambda = \left(\frac{A\mu}{2\hbar^2(-E)}\right)^{1/2} \quad (41)$$

then Equation (39) can be re-written in the following form

$$\frac{d^2R}{d\rho^2} + \frac{1}{\rho}\frac{dR}{d\rho} - \frac{m^2}{\rho^2}R + \frac{\lambda}{\rho}R - \frac{1}{4}R = 0 \quad (42)$$

If we seek solutions for $R(\rho)$ in the form $R(\rho) = exp(-\rho/2)\rho^m S(\rho)$ then we obtain the following differential equation for the function $S(\rho)$

$$\frac{d^2S}{d\rho^2}+\left(\frac{2m+1}{\rho}-1\right)\frac{dS}{d\rho}+\left(\frac{\lambda-m-\frac{1}{2}}{\rho}\right)S=0 \tag{43}$$

Equation (43) can be solved by a series expansion of $S(\rho)$ as , $S(\rho)=\sum_{n=0}a_n\rho^n$, with the coefficients a_n satisfying the recursion relation

$$a_{n+1}=\frac{n+m+\frac{1}{2}-\lambda}{(n+1)(n+2m+1)}a_n \tag{44}$$

Then the energy spectrum can be written explicitly in the form

$$E=\frac{A^2\mu}{2^{\ 2}\left(n+m+\frac{1}{2}\right)^2} \tag{45}$$

It is seen that if the result given in Equation (45) can also be applied to elementary particles which are assumed to behave like a hydrogen-like atom, which is viewed as a two-dimensional physical system, then the intrinsic angular momentum m must take half-integral values.

Now, we show that the wave equation for two-dimensional space given in Equation (20) can also be derived from Dirac equation that describes a quantum particle of half-integer spin. In our previous works [7,8,9], we have shown that both Dirac equation and Maxwell field equations can be formulated from a system of linear first order partial differential equations. Except for the dimensions that involve with the field equations, the formulations of Dirac and Maxwell field equations are remarkably similar and a prominent feature that arises from the formulations is that the equations are formed so that the components of the wavefunctions satisfy a wave equation. However, there are essential differences between the physical interpretations of Dirac and Maxwell physical fields. On the one hand, Maxwell electromagnetic field is a classical field which is composed of two different fields that have different physical properties even though they can be converted into each other. On the other hand, despite Dirac field was originally formulated to describe the dynamics of a single particle, such as the electron, it turned out that a solution to Dirac equation describes not only the dynamics of the electron with positive energy but it also describes the dynamics of the same electron with negative energy. The difficulty that is related to the negative energy can be resolved if the negative energy solutions can be identified as positive energy solutions that can be used to describe the dynamics of a positron. The seemingly confusing situation suggests that Dirac field of massive particles may actually be composed of two physical fields, similar to the case of the electromagnetic field which is composed of the electric field and the magnetic field. Dirac equation can be derived from Equation (12) by imposing the following conditions on the matrices A_i

$$A_i^2=\pm 1 \tag{46}$$

$$A_iA_j+A_jA_i=0 \quad \text{for} \quad i\neq j \tag{47}$$

For the case of $n=4$, the matrices A_i can be shown to take the form

$$A_1=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\quad A_2=\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{pmatrix}\quad A_3=\begin{pmatrix}0&0&0&-i\\0&0&i&0\\0&i&0&0\\-i&0&0&0\end{pmatrix}\quad A_4=\begin{pmatrix}0&0&1&0\\0&0&0&-1\\-1&0&0&0\\0&1&0&0\end{pmatrix} \tag{48}$$

With , $k_1=m, \sigma=1$ and $k_2=0$, the system of linear first order partial differential equations given in Equation (12) reduces to Dirac equation [23]

$$-\frac{\partial\psi_1}{\partial t}-im\psi_1=\left(\frac{\partial}{\partial x}-i\frac{\partial}{\partial y}\right)\psi_4+\frac{\partial\psi_3}{\partial z} \tag{49}$$

$$-\frac{\partial\psi_2}{\partial t}-im\psi_2=\left(\frac{\partial}{\partial x}+i\frac{\partial}{\partial y}\right)\psi_3-\frac{\partial\psi_4}{\partial z} \tag{50}$$

$$\frac{\partial\psi_3}{\partial t}-im\psi_3=\left(-\frac{\partial}{\partial x}+i\frac{\partial}{\partial y}\right)\psi_2-\frac{\partial\psi_1}{\partial z} \tag{51}$$

$$\frac{\partial\psi_4}{\partial t}-im\psi_4=\left(-\frac{\partial}{\partial x}-i\frac{\partial}{\partial y}\right)\psi_1+\frac{\partial\psi_2}{\partial z} \tag{52}$$

With the form of the field equations given in Equations (49-52), we may interpret that the change of the field (ψ_1,ψ_2) with respect to time generates the field (ψ_3,ψ_4), similar to the case of Maxwell field equations in which the change of the electric field generates the magnetic field. With this observation it may be suggested that, like the Maxwell electromagnetic field which is composed of two essentially different physical fields, the Dirac field of massive particles may also be viewed as being composed of two different physical fields, namely the field (ψ_1,ψ_2), which plays the role of the electric field in Maxwell field equations, and the field

(ψ_1, ψ_2), which plays the role of the magnetic field. The similarity between Maxwell field equations and Dirac field equations can be carried further by showing that it is possible to reformulate Dirac equation as a system of real equations. When we formulate Maxwell field equations from a system of linear first order partial differential equations we rewrite the original Maxwell field equations from a vector form to a system of first order partial differential equations by equating the corresponding terms of the vectorial equations. Now, since, in principle, a complex quantity is equivalent to a vector quantity therefore in order to form a system of real equations from Dirac complex field equations we equate the real parts with the real parts and the imaginary parts with the imaginary parts. In this case Dirac equation given in Equations (49-52) can be rewritten as a system of real equations as follows

$$\frac{\partial \psi_1}{\partial t} + \frac{\partial \psi_4}{\partial x} + \frac{\partial \psi_3}{\partial z} = 0 \tag{53}$$

$$\frac{\partial \psi_2}{\partial t} + \frac{\partial \psi_3}{\partial x} - \frac{\partial \psi_4}{\partial z} = 0 \tag{54}$$

$$\frac{\partial \psi_3}{\partial t} + \frac{\partial \psi_2}{\partial x} + \frac{\partial \psi_1}{\partial z} = 0 \tag{55}$$

$$\frac{\partial \psi_4}{\partial t} + \frac{\partial \psi_1}{\partial x} - \frac{\partial \psi_2}{\partial z} = 0 \tag{56}$$

$$\frac{\partial \psi_4}{\partial y} = m\psi_1 \tag{57}$$

$$-\frac{\partial \psi_3}{\partial y} = m\psi_2 \tag{58}$$

$$-\frac{\partial \psi_2}{\partial y} = m\psi_3 \tag{59}$$

$$\frac{\partial \psi_1}{\partial y} = m\psi_4 \tag{60}$$

The system of Dirac field equation given in Equations (53-60) can be considered as a particular case of a more general system of field equations written in the matrix form

$$\left(A_1 \frac{\partial}{\partial t} + A_2 \frac{\partial}{\partial x} + A_3 \frac{\partial}{\partial y} + A_4 \frac{\partial}{\partial z}\right)\psi = m\psi \tag{61}$$

where $\psi = (\psi_1, \psi_2, \psi_3, \psi_4)^T$ and the real matrices A_i are given as

$$A_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad A_2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad A_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad A_4 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \tag{62}$$

The matrices A_i satisfy the following commutation relations

$$A_i^2 = 1 \qquad \text{for} \qquad i = 1, 2, 3, 4 \tag{63}$$

$$A_1 A_i + A_i A_1 = 2A_i \qquad \text{for} \qquad i = 2, 3, 4 \tag{64}$$

$$A_2 A_3 + A_3 A_2 = 2\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{65}$$

$$A_2 A_4 + A_4 A_2 = 0 \tag{66}$$

$$A_3 A_4 + A_4 A_3 = 2\begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{67}$$

By applying $\left(A_1\frac{\partial}{\partial t}+A_2\frac{\partial}{\partial x}+A_3\frac{\partial}{\partial y}+A_4\frac{\partial}{\partial z}\right)$ to Equation (61) and using the commutation relations given in Equations (63-67), then it can be shown that all components of the wavefunction $\psi=(\psi_1,\psi_2,\psi_3,\psi_4)^T$ satisfy the following equation

$$\frac{\partial^2\psi_i}{\partial t^2}-\frac{\partial^2\psi_i}{\partial x^2}+\frac{\partial^2\psi_i}{\partial y^2}-\frac{\partial^2\psi_i}{\partial z^2}=m^2\psi_i \quad \text{for} \quad i=1,2,3,4 \tag{68}$$

If the wavefunction ψ satisfies Dirac field equations given in Equations (53-60) then we obtain the following system of equations for all components

$$\frac{\partial^2\psi_i}{\partial^2 y}-m^2\psi_i=0 \tag{69}$$

$$\frac{\partial^2\psi_i}{\partial t^2}-\frac{\partial^2\psi_i}{\partial x^2}-\frac{\partial^2\psi_i}{\partial z^2}=0 \tag{70}$$

Solutions to Equation (69) are

$$\psi_i=c_1(x,z)e^{my}+c_2(x,z)e^{-my} \tag{71}$$

where c_1 and c_2 are undetermined functions of (x,z), which may be assumed to be constant. The solutions given in Equation (71) give a distribution of a physical quantity, such as the mass of a quantum particle, along the *y*-axis. On the other hand, Equation (70) can be used to describe the dynamics, for example, of a vibrating membrane in the (x,z) - plane. Solutions to Equation (70) can also be found in the form given in Equation (20). Even though elementary particles may have the geometric and topological structures of a 3D differentiable manifold, it is seen from the above descriptions via the Schrödinger wave equation and Dirac equation that they appear as 3D physical objects that embedded in three-dimensional Euclidean space. In Section 5 we will show that this may not be the case for elementary particles of integer spin, such as photons. However, in the next section we will show that the appearance of elementary particles of half-integer spin as 3D physical objects can be justified further by considering other physical properties that are associated with them, such as charge and magnetic monopole.

IV. On The Electric Charge And Magnetic Monopole

In Section 3 we show that massive quantum particles of half-integer spin can be described as 2D differentiable manifolds which are endowed with the geometric and topological structure similar to that of a gyroscope whose main component is a rotating and vibrating membrane that can be described by the solutions of a two-dimensional wave equation, in particular a 2D wave equation that is derived from the Dirac equation of relativistic quantum mechanics. However, the dynamics of the quantum particle is associated only with the distribution of mass of the particle but not other equally important physical matter, such as charge and magnetic monopole. In this section we will discuss further these physical properties of a quantum particle and show that they may be associated with the topological structure of the particle rather than physical quantities that form or are contained inside the particle. As shown in our works on the principle of least action and spacetime structures of quantum particles, the charge of a physical system may depend on the topological structure of the system and is classified by the homotopy group of closed surfaces [24]. In quantum mechanics, the Feynman's method of sum over random paths can be extended to higher-dimensional spaces to formulate physical theories in which the transition amplitude between states of a quantum mechanical system is the sum over random hypersurfaces [25]. This generalisation of the path integral method in quantum mechanics has been developed and applied to other areas of physics, such as condensed matter physics, quantum field theories and quantum gravity theories, mainly for the purpose of field quantisation. In the following, however, we focus attention on the general idea of a sum over random surfaces. This formulation is based on surface integral methods by generalising the differential formulation as discussed for the Bohr's model of a hydrogen-like atom. Consider a surface in R^3 defined by the relation $x^3=f(x^1,x^2)$. The Gaussian curvature K is given by the relation $K=(f_{11}f_{22}-(f_{12})^2)/(1+f_1^2+f_2^2)^2$, where $f_\mu=\partial f/\partial x^\mu$ and $f_{\mu\nu}=\partial^2 f/\partial x^\mu\,\partial x^\nu$, [18]. Let P be a three-dimensional physical quantity which plays the role of the momentum p in the two-dimensional space action integral. The quantity P can be identified with the surface density of a physical quantity, such as charge. Since the momentum p is proportional to the curvature κ, which determines the planar path of a particle, it is seen that in the three-dimensional space the quantity P should be proportional to the Gaussian curvature K, which is used to characterise a surface. If we consider a surface action integral of the form $S=\int PdA=\int(q_e/2\pi)KdA$, where q_e is a universal constant, which plays the role of Planck's constant, then we have

$$S=\frac{q_e}{2\pi}\int\frac{f_{11}f_{22}-(f_{12})^2}{(1+f_1^2+f_2^2)^{3/2}}dx^1dx^2 \tag{72}$$

According to the calculus of variations, similar to the case of path integral, to extremise the action integral $S = \int L(f, f_\mu, f_{\mu\nu}, x^\mu)dx^1dx^2$, the functional $L(f, f_\mu, f_{\mu\nu}, x^\mu)$ must satisfy the differential equations [26]

$$\frac{\partial L}{\partial f} - \frac{\partial}{\partial x^\mu}\frac{\partial L}{\partial f_\mu} + \frac{\partial^2}{\partial x^\mu \partial x^\nu}\frac{\partial L}{\partial f_{\mu\nu}} = 0 \qquad (73)$$

It can be verified that with the functional of the form given in Equation (72) the differential equation given by Equation (73) is satisfied by any surface. Hence, we can generalise Feynman's postulate of random path to formulate a quantum theory in which the transition amplitude between states of a quantum mechanical system is a sum over random surfaces, provided the functional P in the action integral $S = \int PdA$ is taken to be proportional to the Gaussian curvature K of a surface. Consider a closed surface and assume that we have many such different surfaces which are described by the higher dimensional homotopy groups. As in the case of the fundamental homotopy group of paths, we choose from among the homotopy class a representative spherical surface, in which case we can write

$$\oint PdA = \frac{q_e}{4\pi}\oint d\Omega, \qquad (74)$$

where $d\Omega$ is an element of solid angle. Since $\oint d\Omega$ depends on the homotopy class of the spheres that it represents, we have $\oint d\Omega = 4\pi n$ where n is the topological winding number of the higher dimensional homotopy group. From this result we obtain a generalised Bohr quantum condition

$$\oint PdA = nq_e \qquad (75)$$

From the result obtained in Equation (75), as in the case of Bohr's theory of quantum mechanics, we may consider a quantum process in which a physical entity transits from one surface to another with some radiation-like quantum created in the process. Since this kind of physical process can be considered as a transition from one homotopy class to another, the radiation-like quantum may be the result of a change of the topological structure of the physical system, and so it can be regarded as a topological effect. Furthermore, it is interesting to note that the action integral $(q_e/4\pi)\oint KdA$ is identical to Gauss's law in electrodynamics [27]. In this case the constant q can be identified with the charge of a particle, which represents the topological structure of a physical system and the charge of a physical system must exist in multiples of q_e. Hence, the charge of a physical system may depend on the topological structure of the system and is classified by the homotopy group of closed surfaces. This result may shed some light on why charge is quantised even in classical physics. As a further remark, we want to mention here that in differential geometry, the Gaussian is related to the Ricci scalar curvature R by the relation $R = 2K$. And it has been shown that the Ricci scalar curvature can be identified with the potential of a physical system, therefore our assumption of the existence of a relationship between the Gaussian curvature and the surface density of a physical quantity can be justified [1]. Now, in order to establish a relationship between the electric charge q_e and the magnetic monopole associated with a quantum particle, similar to Dirac relation $\hbar c/q_e q_m = 2$, we need to extend Feynman's method of sum over random surfaces to temporal dynamics in which the magnetic monopole can also be considered as a topological structure of a temporal continuum. Even though the following results are similar to those obtained for the spatial Euclidean continuum, for clarity, we will give an abbreviated version by first defining a temporal Gaussian curvature in the temporal Euclidean continuum R^3 and then deriving a quantised magnetic charge from Feynman integral method. As in spatial dimensions, we consider a temporal surface defined by the relation $t^3 = f(t^1, t^2)$. Then, as shown in differential geometry, the temporal Gaussian curvature denoted by K_T can be determined by $f(t^1, t^2)$ and given as $K_T = (f_{11}f_{22} - (f_{12})^2)/(1+f_1^2+f_2^2)^2$, where $f_\mu = \partial f/\partial t^\mu$ and $f_{\mu\nu} = \partial^2 f/\partial t^\mu \partial t^\nu$. Let P_T be a 3-dimensional physical quantity which will be identified with the surface density of a magnetic substance, such as magnetic charge of an elementary particle. We therefore assume that an elementary particle is assigned not only with an electric charge q_e but also a magnetic charge q_m. We further assume that the quantity P_T is proportional to the temporal Gaussian curvature K_T. Now, as in the case with spatial dimensions, if we consider a surface action integral of the form $S = \int P_T dA_T = \int (q_m/2\pi) K_T dA_T$, then we have

$$S = \frac{q_m}{2\pi}\int \frac{f_{11}f_{22} - (f_{12})^2}{(1+f_1^2+f_2^2)^{3/2}}dt^1dt^2 \qquad (76)$$

Similar to the case of the spatial integral, to extremise the action integral given in Equation (76), the functional $L(f, f_\mu, f_{\mu\nu}, t^\mu)$ must satisfy the differential equation given in Equation (73). Hence, we can also generalise Feynman's postulate of random surfaces to formulate a quantum theory in which the transition amplitude between states of a quantum mechanical system is a sum over random surfaces, provided the functional P_T in the action integral $S = \int P_T dA_T$ is taken to be proportional to the temporal Gaussian curvature K_T of a temporal surface. Similar to the random spatial surfaces, we obtain the following result

$$\oint P_T dA_T = \frac{q_m}{4\pi}\oint d\Omega = n_T q_m \tag{77}$$

The action integral $(q_m/4\pi)\oint K_T\, dA_T$ is similar to Gauss's law in electrodynamics. In this case the constant q_m can be identified with the magnetic charge of a particle. In particular, the magnetic charge q_m represents the topological structure of a physical system must exist in multiples of q_m Hence, the magnetic charge of a physical system, such as an elementary particle, may depend on the topological structure of the system and is classified by the homotopy group of closed surfaces. We are now in the position to show that it is possible to obtain the relationship between the electric charge q_e and the magnetic charge q_m derived by Dirac by considering a spatiotemporal Gaussian curvature K which is defined as a product of the temporal Gaussian curvature K_T and the spatial Gaussian curvature K_S as follows

$$K = K_T \times K_S \tag{78}$$

The spatiotemporal submanifold that gives rise to this form of curvature is homeomorphic to $S^2 \times S^2$ If K_T and K_S are independent from each other then we can write

$$\oint K\, dA = \oint K_T \times K_S\, dA_T dA_S = \oint K_T\, dA_T \times \oint K_S\, dA_S \tag{79}$$

If we assume further that $\oint K\, dA = k$, where k is an undetermined constant, then using the results $\oint K_S\, dA_S = \oint (q_e/2\pi) K_S\, dA_S$ and $\oint K_T\, dA_T = \oint (q_m/2\pi) K_T\, dA_T$, we obtain a general relationship between the electric charge q_e and the magnetic charge q_m

$$\frac{k}{q_e q_m} = n_S n_T \tag{80}$$

In particular, if $n_S = 1$, $n_T = 2$ and $k = \hbar c$, or $n_S = 2$ $n_T = 1$ and $k = \hbar c$, then we recover the relationship obtained by Dirac, $\hbar c/q_e q_m = 2$.

In the classical electromagnetic field, Maxwell field equations describe a conversion between the electric and magnetic field. If the electric field is associated with the electric charge, which is in turn associated with the spatial continuum, and the magnetic field with the magnetic charge, which is in turn associated with the temporal continuum, then we may speculate that the electromagnetic field is a manifestation of a conversion between the spatial and temporal manifolds. In the following we show that if we consider the spatiotemporal manifold as a spherical fiber bundle then it is possible to describe the electromagnetic field as a wave through a medium of fibers that are composed of 3-spheres [14, 28]. In classical physics, the formation of a wave requires a medium which is a collection of physical objects therefore with this classical picture in mind we may assume that the medium for the electromagnetic and matter waves is composed of quantum particles which have the geometric and topological structures of spatiotemporal *n*-cells that are decomposed from the spatiotemporal manifold at each point of the spatiotemporal continuum. This is equivalent to considering the spatiotemporal manifold as a fiber bundle $E = B \times F$, where B is the base space, which is the spatiotemporal continuum, and the fiber F, which is the *n*-cells. In the following we will only consider an *n*-cell as an *n*-sphere S^n and the total spatiotemporal manifold M will be regarded as an *n*-sphere bundle. It is reasonable to suggest that there may exist physical fields that are associated with different dimensions of the *n*-spheres, however, as an illustration, we will consider only the case with $n=6$ so that S^6 is homeomorphic to $S_S^3 \times S_T^3$, hence the medium of the electromagnetic and matter waves will be assumed to be composed of $S_S^3 \times S_T^3$ cells at each point of the spatiotemporal manifold. In other words, the 6-sphere fibers form the required medium for the electromagnetic and matter waves. Consequently, the problem that we want to address reduces to the problem of the conversion between the spatial and temporal manifolds S_S^3 and S_T^3. It is expected that the formulation of such conversion should be derived from a general line element $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$. As examples, we will show in the following that the conversion between the spatial and temporal manifolds S_S^3 and S_T^3 can be described by assuming the general line element to take the form of either a centrally symmetric metric or the Robertson-Walker metric [29]. A general six-dimensional centrally symmetric metric can be written as

$$ds^2 = e^{\psi}c^2dt^2 + c^2t^2\left(d{\theta_T}^2 + \sin^2\theta_T d{\phi_T}^2\right) - e^{\chi}dr^2 - r^2\left(d{\theta_S}^2 + \sin^2\theta_S d{\phi_S}^2\right) \tag{81}$$

If we rearrange the (θ, ϕ) directions of both the spatial and the temporal cells so that they coincide $\theta_S = \theta_T = \theta$, and $\phi_S = \phi_T = \phi$ then we have

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 - (r^2 - c^2t^2)(d\theta^2 + \sin^2\theta d\phi^2) \tag{82}$$

There are profound differences in the structure of the spatiotemporal manifold that arise from the line element given in Equation (82). The line element in Equation (82) can be rewritten in the form

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 - r^2\left(1-\frac{c^2}{v^2}\right)(d\theta^2+\sin^2\theta d\phi^2) \tag{83}$$

where we have defined the new quantity that has the dimension of speed as $v = r/t$. It is seen that if $v > c$ then the line element given in Equation (83) can lead to the conventional structure of spacetime in which, effectively, space has three dimensions and time has one dimension, and that if $v < c$ then the line element given in Equation (83) can lead to the conventional structure of spacetime in which time has three dimensions and space has one. However, for the purpose of discussing a conversion between the temporal manifold and the spatial manifold of spacetime we would need to consider possible relationship between space and time and how they change with respect to each other continuously. In order to fulfil this task we need to utilise the results obtained in our works on geometric interactions that show that there are various forces associated with the decomposed *n*-cells from which, by applying Newton's law of dynamics, different possible relationships between space and time could be derived [12,13]. For example, by applying the temporal Newton's second law for radial motion to the force that is associated with decomposed 1-cells we obtain

$$D\frac{d^2t}{dr^2} = {}_1t \tag{84}$$

General solutions to the equation given in Equation (84) are

$$t = c_1e^{\sqrt{h_1/D}r} + c_2e^{-\sqrt{h_1/D}r} \tag{85}$$

If $D = -m$ and $h_1 > 0$ then the following solution can be obtained

$$t = A\sin(\omega r) \tag{86}$$

where $\omega = \sqrt{h_1/D}$. By differentiation we have

$$\frac{dt}{dr} = A\omega\cos(\omega r) \tag{87}$$

If we assume a linear approximation between space and time for the values of $v \sim c$, i.e., $dr/dt \sim r/t = v$ then Equation (83) becomes

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 - r^2\left(1-c^2A^2\omega^2\cos^2(\omega r)\right)(d\theta^2+\sin^2\theta d\phi^2) \tag{88}$$

It is seen from Equation (88) that if $1-c^2A^2\omega^2\cos^2(\omega r)>0$ then effectively spacetime appears as a spatial manifold in which there are three spatial dimensions and one temporal dimension. Therefore it is expected that for $1-c^2A^2\omega^2\cos^2(\omega r)<0$ spacetime would appear as a temporal manifold. This is in fact the case as can be shown as follows. Instead of the metric form given in Equation (83), the line element given in Equation (82) can also be re-written in a different form as follows

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 + c^2t^2\left(1-\frac{v^2}{c^2}\right)(d\theta^2+\sin^2\theta d\phi^2) \tag{89}$$

Using Equation (87) we obtain

$$ds^2 = e^{\psi}c^2dt^2 + c^2t^2\left(1-\frac{1}{c^2A^2\omega^2\cos^2(\omega r)}\right)(d\theta^2+\sin^2\theta d\phi^2) - e^{\chi}dr^2 \tag{90}$$

Therefore, if the condition $1-c^2A^2\omega^2\cos^2(\omega r)<0$ is satisfied then Equation (90) is reduced to a line element for the spatiotemporal manifold which effectively has three temporal dimensions and one spatial dimension.

For the case $r^2 - c^2t^2 \neq 0$ the line element given in Equation (82) can be determined by applying Einstein field equations of general relativity

$$ds^2 = \left(1-\frac{2GM}{c^2r}\right)c^2dt^2 - \left(1-\frac{2GM}{c^2r}\right)^{-1}dr^2 - (r^2-c^2t^2)(d\theta^2+\sin^2\theta d\phi^2) \tag{91}$$

It should also be mentioned here that for the case $r^2 - c^2t^2 = 0$, the line element given in Equation (82) reduces to the simple form

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 \tag{92}$$

and as discussed in our previous works that spacetime that is endowed with this particular metric appears to behave as a wave where the functions ψ and χ satisfy the wave equation

$$\frac{\partial^2\psi}{c^2\partial t^2} - \frac{\partial^2\psi}{\partial r^2} = 0 \tag{93}$$

We can also obtain a conversion between the spatial and temporal manifolds similar to those that have been discussed above if we use the spatial Newton's second law instead. In this case the following results can be obtained

$$m\frac{d^2r}{dt^2} = k_1 r \tag{94}$$

$$r = c_1 e^{\sqrt{k_1/m}t} + c_2 e^{-\sqrt{k_1/m}t} \tag{95}$$

If we consider the case $m > 0$ and $k_1 < 0$ then we can obtain a simple solution

$$r = A\sin(\omega t) \tag{96}$$

where $\omega = \sqrt{-k_1/m}$. By differentiation, we obtain

$$\frac{dr}{dt} = A\omega\cos(\omega t) \tag{97}$$

If we also assume a linear approximation between space and time for the values of $v \sim c$, i.e., $dr/dt \sim r/t = v$ then then the line elements become

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 - r^2\left(1 - \frac{c^2}{A^2\omega^2\cos^2(\omega t)}\right)(d\theta^2 + \sin^2\theta d\phi^2) \tag{98}$$

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 + c^2t^2\left(1 - \frac{A^2\omega^2\cos^2(\omega t)}{c^2}\right)(d\theta^2 + \sin^2\theta d\phi^2) \tag{99}$$

It is seen from Equations (98) and (99) that there is also a conversion between the spatial and temporal submanifolds of the 6-spherical cells that are decomposed from the total spatiotemporal manifold.

Now, we consider the case when the decomposed $S_S^3 \times S_T^3$ cells from the spatiotemporal manifold are furnished with the Robertson-Walker metric. In the spatiotemporal manifold which has three spatial dimensions and one temporal dimension, the Robertson-Walker metric is given as

$$ds^2 = c^2dt^2 - S^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) \tag{100}$$

With the decomposition of $S_S^3 \times S_T^3$ cells from the spatiotemporal manifold which has the mathematical structure of an *n*-sphere bundle, the Robertson-Walker metric is assumed to be extended to a six-dimensional line element of the form

$$ds^2 = S^2(r)\left(\frac{dt^2}{1-k_Tt^2} + t^2\left(d{\theta_T}^2 + \sin^2\theta_T d{\phi_T}^2\right)\right)$$
$$- S^2(t)\left(\frac{dr^2}{1-k_Sr^2} + r^2\left(d{\theta_S}^2 + \sin^2\theta_S d{\phi_S}^2\right)\right) \tag{101}$$

If we also arrange the (θ, ϕ) directions of both spatial and the temporal manifolds so that $\theta_S = \theta_T = \theta$ and $\phi_S = \phi_T = \phi$ then the general space-time metric given in Equation (101) becomes

$$ds^2 = \frac{S^2(r)dt^2}{1-k_T t^2} - \frac{S^2(t)dr^2}{1-k_S r^2} - \left(r^2S^2(t) - t^2S^2(r)\right)(d\theta^2 + \sin^2\theta d\phi^2) \tag{102}$$

Equation (102) can be rewritten in the following form

$$ds^2 = \frac{S^2(r)dt^2}{1-k_T t^2} - \frac{S^2(t)dr^2}{1-k_S r^2} - \left(S^2(t) - \frac{1}{v^2}S^2(r)\right)r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{103}$$

where we have also defined $v = r/t$. Now, we need to look for possible relationships between space and time so that they can show a conversion between the temporal component S_T^3 and the spatial component S_S^3 of the decomposed spatiotemporal cells $S_S^3 \times S_T^3$. Even though the conditions that will be imposed are rather arbitrarily they do show that the temporal manifold S_T^3 and the spatial manifold S_S^3 can actually be converted into one another. It should also be mentioned that these are not the only conditions that can give rise to a conversion between space and time and, as shown in our works on Euclidean relativity, Euclidean special relativity also produces such conversion [30]. Now, if we impose the following condition

$$\frac{S^2(r)}{1-k_T t^2} = c^2 \tag{104}$$

then the line element given in Equation (103) reduces to

$$ds^2 = c^2dt^2 - \frac{S^2(t)dr^2}{1-k_S r^2} - \left(S^2(t) - \frac{c^2}{v^2}(1-k_T t^2)\right)r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{105}$$

Equation (105) describes particular structures of the temporal manifold with respect to the change of the spatial manifold. Using a linear approximation between space and time for the values of $v \sim c$, then from the relation $1/v^2 = A^2\omega^2\cos^2(\omega r)$, Equation (105) becomes

$$ds^2 = c^2dt^2 - \frac{S^2(t)dr^2}{1-k_S r^2} - S^2(t)\left(1 - \frac{c^2A^2\omega^2\cos^2(\omega r)(1-k_T t^2)}{S^2(t)}\right)r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{106}$$

If we further impose the condition

$$\frac{S^2(t)}{1-k_T t^2} = 1 \tag{107}$$

then we obtain

$$ds^2 = c^2dt^2 - \frac{S^2(t)dr^2}{1-k_S r^2} - S^2(t)\left(1 - c^2A^2\omega^2\cos^2(\omega r)\right)r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{108}$$

It is seen from the line element given in Equation (108) that if $\left(1 - c^2A^2\omega^2\cos^2(\omega r)\right) > 0$ then effectively the spatiotemporal manifold behaves as a spatial manifold endowed with the Robertson-Walker metric. On the other hand, the six-dimensional Robertson-Walker metric can also be written as

$$ds^2 = \frac{S^2(r)dt^2}{1-k_T t^2} - \frac{S^2(t)dr^2}{1-k_S r^2} - \left(v^2S^2(t) - S^2(r)\right)t^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{109}$$

If we impose the following condition

$$\frac{S^2(t)}{1-k_S r^2} = {c_T}^2 \tag{110}$$

then we obtain

$$ds^2 = \frac{S^2(r)dt^2}{1-k_T t^2} - {c_T}^2 dr^2 + S^2(r)\left(1 - \frac{v^2 {c_T}^2(1-k_S r^2)}{S^2(r)}\right) t^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (111)$$

From the linear approximation $1/v^2 \sim A^2\omega^2\cos^2(\omega r)$, Equation (111) becomes

$$ds^2 = \frac{S^2(r)dt^2}{1-k_T t^2} - {c_T}^2 dr^2 + S^2(r)\left(1 - \frac{{c_T}^2(1-k_S r^2)}{A^2\omega^2\cos^2(\omega r)S^2(r)}\right) t^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (112)$$

If we further impose the condition

$$\frac{S^2(r)}{1-k_S r^2} = {c_T}^2 c^2 \quad (113)$$

then we obtain

$$ds^2 = \frac{S^2(r)dt^2}{1-k_T t^2} - {c_T}^2 dr^2 + S^2(r)\left(1 - \frac{1}{c^2A^2\omega^2\cos^2(\omega r)}\right) t^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (114)$$

Therefore if $\left(1 - c^2A^2\omega^2\cos^2(\omega r)\right) < 0$ then effectively the spatiotemporal manifold behaves as a temporal manifold endowed with the temporal Robertson-Walker metric

$$ds^2 = S^2(r)\left(\frac{dt^2}{1-k_T t^2} + t^2(d\theta^2 + \sin^2\theta d\phi^2)\right) - {c_T}^2 dr^2 \quad (115)$$

It is also noted from the line element given in Equation (102) that when space and time satisfy the condition $r^2S^2(t) - t^2S^2(r) = 0$ then we have

$$ds^2 = \frac{S^2(r)dt^2}{1-k_T t^2} - \frac{S^2(t)dr^2}{1-k_S r^2} \quad (116)$$

The metric given in Equation (116) is a particular form of the general line element given in Equation (92) with $S^2(r)/(1-k_T t^2) = e^{\psi}c^2$ and $S^2(t)/(1-k_S r^2) = e^{\chi}$, therefore the wave motion of spacetime which is endowed with the Roberson-Walker metric also occurs at the position of conversion between the temporal and spatial manifolds.

V. Quantum Particles With Integer Spin

In Sections 3 and 4 we show that a complete picture of quantum particles can be visualised in the three-dimensional Euclidean space if their associated differentiable manifolds are solutions of a two-dimensional wave equation, and these massive quantum particles have half-integer spin therefore they can be identified with fermions. Actually, the energy spectrum obtained from the Schrödinger wave equation in a two-dimensional space given in Equation (45) also suggests that there may be massive quantum particles of integer spin associated with differentiable manifolds that are solutions of a two-dimensional wave equation. Nonetheless, it has been shown that quantum particles with integer spin, such as the massless quantum particles of the electromagnetic field, are described by a three-dimensional wave equation, therefore it is reasonable to suggest that the differentiable manifolds that are associated with these quantum particles, called bosons, not only should have different geometric and topological structures but also render different perceptions with regard to our observation of their physical behaviour. In classical physics, the dynamics of physical phenomena can be formulated based on the notion of elementary particles that exist as 3D solid balls containing all physical entities that are needed for

physical formulations, such as mass and charge. It is then simply assumed that in order to interact these solid balls somehow generate physical fields, such as the gravitational field and the electromagnetic field, which can be derived from a three-dimensional wave equation. Despite with the fact that the existence of these physical fields is self-evident and they are widely applied their true natures are very much still unknown. However, in quantum physics bosons are quantum particles therefore as in the case of fermions considered in the previous sections we may suggest that bosons also possess the geometric and topological structures of differentiable manifolds which are solutions of a wave equation. Along the line of Einstein's perception of physical existence in which a 3-sphere can be constructed from a four-dimensional Euclidean space R^4, in this section we will discuss the possibility to extend the notion of wave motion into a fourth spatial dimension so that we can have a unified dynamical description in terms of wave equations for quantum particles of any spin. With this in mind, in this section we discuss a spacetime in which space has four dimensions and time has one dimension. Despite a spatial space with four dimensions is simply a mathematical extension of the concept of a spatial space with three dimensions it is still considered to be rather speculative in contrast to the three-dimensional space which is a direct application from the observation from physical existence that we can perceive. In classical physics, the three-dimensional wave equation written in Cartesian coordinates (x,y,z) of the form

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}-\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial y^2}-\frac{\partial^2\psi}{\partial z^2}=0 \qquad (117)$$

can be used to describe the wave motion of different physical fields. However, if we want to generalise the above discussions for 2D wave equations that describe a vibrating membrane then what geometrical characteristic should we assign to the wavefunction ψ? Since in 2D wave equations, the wavefunction are the actual height of the particles that form the medium which can be viewed in the third spatial dimension of the space in which they are embedded, therefore we may suggest that the wavefunction which is a solution to the wave equation given in Equation (117) should also be given the meaning of the height of the particles that form the medium. However, if we want to give the meaning of the height to the 3D wavefunction then the space in which the 3D vibrating object is embedded must be extended to a four-dimensional Euclidean space. Whether such extension can be justified is a subject that requires further investigation and in fact this can be shown to be related to the fundamental question of why we exist as 3D physical objects. Now, consider a region D which is embedded in a three-dimensional Euclidean space and bounded by a closed surface. As in the case of the membrane considered above, we assume that the D region is a physical object that is made up of mass points joined together by contact forces so that it can vibrate. In general, the region D can be any shape, however, as an illustration, we consider a simple case of which the region D is a solid ball embedded in the (x,y,z)-space defined by the relation $D=\{x^2+y^2+z^2<a^2\}$with the condition $\psi=0$ on the boundary of D. In a three-dimensional Euclidean space, such physical objects can only be assumed to vibrate internally inside the solid ball and the mathematical object represented by the function ψ can only be assumed to be a physical entity, such as fluids and acoustics. However, as in the case of the membrane considered in Section 3 in which the mass points of the membrane can vibrate into the third dimension of the three-dimensional Euclidean space, we may assume that the mass points that form the physical object contained in the three-dimensional region $D=\{x^2+y^2+z^2<a^2\}$ can vibrate into the fourth dimension of a four-dimensional Euclidean space, therefore the mathematical object ψ represents a spatial dimension. When vibrating, at each moment of time, the solid ball becomes a three-dimensional differentiable manifold that is embedded in a four-dimensional Euclidean space. In this case, an observer who is a 3D physical object can only observe the cross-section which is the intersection of the time-dependent differentiable manifold and the three-dimensional Euclidean space into which that the observer is embedded. And the cross-section appears as a 3D wave to the 3D observer. Written in the spherical polar coordinates, which are defined in terms of the Cartesian coordinates (x,y,z) as $x=r\sin\theta\cos\phi$, $y=r\sin\theta\sin\phi$, $z=r\cos\theta$ the three-dimensional wave equation given in Equation (117) becomes

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}-\frac{\partial^2\psi}{\partial r^2}-\frac{2}{r}\frac{\partial\psi}{\partial r}-\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)-\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}=0 \qquad (118)$$

The general solution to Equation (118) for the vibrating solid ball with a given initial condition can be found by separating the variables in the form $\psi(r,\theta,\phi,t)=S(r,\theta,\phi)T(t)$

$$\psi(r,\theta,\phi,t)=\sum_{l=0}^{\infty}\sum_{j=1}^{\infty}\sum_{m=-l}^{l}A_{lmj}e^{-k\lambda_{lj}t}\frac{J_{l+\frac{1}{2}}\left(\sqrt{\lambda_{lj}}r\right)}{\sqrt{r}}P_l^m(\cos\theta)\,e^{im\phi} \qquad (119)$$

where $P_l^m(\cos\theta)$ is the associated Legendre function and $J_{l+\frac{1}{2}}(\sqrt{\lambda_{lj}}r)$ is the Bessel function. The wavefunction given in Equation (119) is the general time-dependent shape of the vibrating solid ball embedded in the four-dimensional Euclidean space. Similar to the vibrating membrane, at each moment of time the vibrating solid ball appears as a 3D differentiable manifold which is a geometric object whose geometric structure can be constructed using the wavefunction given in Equation (119) and can be identified with a quantum particle. Therefore, what we observe as a wave may in fact be a particle and this kind of dual existence may be related to the problem of wave-particle duality we encounter in quantum mechanics. A simpler case is that of a quantum particle that appears as a spherical wave. In this case the wave equation given in Equation (119) reduces to

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}-\frac{\partial^2\psi}{\partial r^2}-\frac{2}{r}\frac{\partial\psi}{\partial r}=0 \qquad (120)$$

The general solution to Equation (120) can be found as

$$\psi(r,t)=\frac{c_1 f(r-ct)+c_2 g(r+ct)}{r} \qquad (121)$$

The above wavefunctions describe the geometric structures of quantum particles as differentiable manifolds embedded in a four-dimensional Euclidean space, therefore, if the Ricci scalar curvature of the vibrating solid ball can be formulated in terms of the wavefunction ψ then the geometric structure of the vibrating solid ball can be determined. Actually we can show how such relation can be realised for the case of the hydrogen atom when the Ricci scalar curvature can be constructed from the Schrödinger wavefunctions in wave machanics [1]. We showed that the scalar potential V can be identified with the Ricci scalar curvature as

$$V=kR \qquad (122)$$

where k is an undetermined dimensional constant. Using the relation between the scalar potential and the Ricci scalar curvature given in Equation (122), we can show that the Ricci scalar curvature can be constructed from the wavefunctions obtained from the Schrödinger wave equation in wave mechanics. In his original works, Schrödinger introduced a new function ψ which is real, single-valued and twice differentiable, through the relation $S=\hbar\ln\psi$, where the action S is defined by $S=\int Ldt$ and L is the Lagrangian defined by $L=T-\varphi$ with T is the kinetic energy and φ is the potential energy [21]. By applying the principle of least action defined in classical dynamics, Schrödinger arrived at the wave equation to describe the stationary state of the hydrogen atom

$$\nabla^2\psi+\frac{2m}{2}\left(E+\frac{kq^2}{r}\right)\psi=0 \qquad (123)$$

Now we show that Schrödinger wavefunction ψ can be used to construct the Ricci scalar curvature associated with the spacetime structures of the quantum states of the hydrogen atom. By using the defined relations $L=dS/dt$, $dS/dt=\partial_t S+\sum_{\mu=1}^{3}\partial_\mu S(dx^\mu/dt)$, $T=m\sum_{\mu=1}^{3}(dx^\mu/dt)^2$ and $\varphi=T-L$, the following relation can be obtained

$$\varphi=m\sum_{\mu=1}^{3}(dx^\mu/dt)^2-\frac{\partial_t\psi+\sum_{\mu=1}^{3}\partial_\mu\psi(dx^\mu/dt)}{\psi} \qquad (124)$$

Using the relations $V=k\mathrm{R}$ and $V=\varphi/m$, we obtain the following relationship between the Schrödinger wavefunction ψ and the Ricci scalar curvature R

$$\mathrm{R}=\frac{1}{k}\left(\sum_{\mu=1}^{3}(dx^\mu/dt)^2-\frac{1}{m}\frac{\partial_t\psi+\sum_{\mu=1}^{3}\partial_\mu\psi(dx^\mu/dt)}{\psi}\right) \qquad (125)$$

Finally, we would like to give more details how to formulate Maxwell field equations from the general system of linear first order partial differential equations given in Equation (12). In order to derive Maxwell field equations from Equation (12) we would need to identify the matrices A_i. For the case of Dirac equation, we simply impose the condition $A_iA_j+A_jA_i=0$ for $i\neq j$ and $A_i^2=1$. However, as shown below, for Maxwell field equations the identification of the matrices A_i is almost impossible without relying on the form of Maxwell field equations that have been established in classical electrodynamics. With the notation $\psi=(E_x,E_y,E_z,B_x,B_y,B_z)^T=(\psi_1,\psi_2,\psi_3,\psi_4,\psi_5,\psi_6)$, and $\epsilon\mu=1$, the most symmetric form of Maxwell field equations of the electromagnetic field that are derived from Faraday's law and Ampere's law can be written as

$$\frac{\partial \psi_1}{\partial t} + \mu j_1 = \frac{\partial \psi_6}{\partial y} - \frac{\partial \psi_5}{\partial z} \quad (126)$$

$$\frac{\partial \psi_2}{\partial t} + \mu j_2 = \frac{\partial \psi_4}{\partial z} - \frac{\partial \psi_6}{\partial x} \quad (127)$$

$$\frac{\partial \psi_3}{\partial t} + \mu j_3 = \frac{\partial \psi_5}{\partial x} - \frac{\partial \psi_4}{\partial y} \quad (128)$$

$$\frac{\partial \psi_4}{\partial t} + j_4 = \frac{\partial \psi_2}{\partial z} - \frac{\partial \psi_3}{\partial y} \quad (129)$$

$$\frac{\partial \psi_5}{\partial t} + j_5 = \frac{\partial \psi_3}{\partial x} - \frac{\partial \psi_1}{\partial z} \quad (130)$$

$$\frac{\partial \psi_6}{\partial t} + j_6 = \frac{\partial \psi_1}{\partial y} - \frac{\partial \psi_2}{\partial x} \quad (131)$$

where $J = (j_1, j_2, j_3, j_4, j_5, j_6)^T$ is the electromagnetic current in which the electric current is $j_e = (j_1, j_2, j_3)$ and the magnetic current is $j_m = (j_4, j_5, j_6)$. The system of equations given in Equations (126-131) can be written the following matrix form

$$\left(A_1 \frac{\partial}{\partial t} + A_2 \frac{\partial}{\partial x} + A_3 \frac{\partial}{\partial y} + A_4 \frac{\partial}{\partial z}\right)\psi = A_5 J \quad (132)$$

with the matrices A_i are given as

$$A_1 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad A_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad A_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad A_5 = \begin{pmatrix} \mu & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (133)$$

Furthermore, if an additional condition that imposes on the function ψ that requires that it also satisfies the wave equation given by Equation (15) then Gauss's laws will be recovered. From Equation (133) we obtain

$$A_1^2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad A_2^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} \quad A_3^2 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$A_4^2 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad A_5^2 = \begin{pmatrix} \mu^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_2A_3 + A_3A_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad A_2A_4 + A_4A_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad A_3A_4 + A_4A_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$A_1A_i + A_iA_1 \quad \text{for} \quad i = 2, 3, 4 \quad (134)$$

Now, if we apply the differential operator $(A_1\partial/\partial t + A_2\partial/\partial x + A_3\partial/\partial y + A_4\partial/\partial z)$ to Equation (132) then we arrive at

$$\left(\begin{pmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{pmatrix}\frac{\partial^2}{\partial t^2}+\begin{pmatrix}0&0&0&0&0&0\\0&-1&0&0&0&0\\0&0&-1&0&0&0\\0&0&0&0&0&0\\0&0&0&0&-1&0\\0&0&0&0&0&-1\end{pmatrix}\frac{\partial^2}{\partial x^2}+\begin{pmatrix}-1&0&0&0&0&0\\0&0&0&0&0&0\\0&0&-1&0&0&0\\0&0&0&-1&0&0\\0&0&0&0&0&0\\0&0&0&0&0&-1\end{pmatrix}\frac{\partial^2}{\partial y^2}\right.$$
$$+\begin{pmatrix}-1&0&0&0&0&0\\0&-1&0&0&0&0\\0&0&0&0&0&0\\0&0&0&-1&0&0\\0&0&0&0&-1&0\\0&0&0&0&0&0\end{pmatrix}\frac{\partial^2}{\partial z^2}+\begin{pmatrix}0&1&0&0&0&0\\1&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&1&0\\0&0&0&1&0&0\\0&0&0&0&0&0\end{pmatrix}\frac{\partial^2}{\partial x\partial y}+\begin{pmatrix}0&0&1&0&0&0\\0&0&0&0&0&0\\1&0&0&0&0&0\\0&0&0&0&0&1\\0&0&0&0&0&0\\0&0&0&1&0&0\end{pmatrix}\frac{\partial^2}{\partial x\partial z}$$
$$\left.+\begin{pmatrix}0&0&0&0&0&0\\0&0&1&0&0&0\\0&1&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&1\\0&0&0&0&1&0\end{pmatrix}\frac{\partial^2}{\partial y\partial z}\right)\psi=-\left(\begin{pmatrix}\mu&0&0&0&0&0\\0&\mu&0&0&0&0\\0&0&\mu&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{pmatrix}\frac{\partial}{\partial t}\right)J \tag{135}$$

From the equation given in Equation (135), using Gauss's law $\nabla.\mathbf{E}=\rho_e/\epsilon$ we obtain the following wave equations for the components of the electric field $\mathbf{E}=\left(E_x,E_y,E_z\right)=(\psi_1,\psi_2,\psi_3)$

$$\frac{\partial^2\psi_i}{\partial t^2}-\frac{\partial^2\psi_i}{\partial x^2}-\frac{\partial^2\psi_i}{\partial y^2}-\frac{\partial^2\psi_i}{\partial z^2}=-\mu\frac{\partial j_i}{\partial t}\quad\text{for}\quad i=1,2,3 \tag{136}$$

Similarly for the magnetic field $\mathbf{B}=\left(B_x,B_y,B_z\right)=(\psi_4,\psi_5,\psi_6)$ we can also obtain the following wave equations for the components of the magnetic field $\mathbf{B}=\left(B_x,B_y,B_z\right)=(\psi_4,\psi_5,\psi_6)$

$$\frac{\partial^2\psi_i}{\partial t^2}-\frac{\partial^2\psi_i}{\partial x^2}-\frac{\partial^2\psi_i}{\partial y^2}-\frac{\partial^2\psi_i}{\partial z^2}=-\frac{\partial j_i}{\partial t}\quad\text{for}\quad i=4,5,6 \tag{137}$$

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FARSS designated members are entitled to avail a 40% discount while publishing their research papers (of a single author) with Global Journals Incorporation (USA), if the same is accepted by Editorial Board/Peer Reviewers. If you are a main author or co-author in case of multiple authors, you will be entitled to avail discount of 10%.

Once FARSB title is accorded, the Fellow is authorized to organize a symposium/seminar/conference on behalf of Global Journal Incorporation (USA). The Fellow can also participate in conference/seminar/symposium organized by another institution as representative of Global Journal. In both the cases, it is mandatory for him to discuss with us and obtain our consent.

You may join as member of the Editorial Board of Global Journals Incorporation (USA) after successful completion of three years as Fellow and as Peer Reviewer. In addition, it is also desirable that you should organize seminar/symposium/conference at least once.

We shall provide you intimation regarding launching of e-version of journal of your stream time to time.This may be utilized in your library for the enrichment of knowledge of your students as well as it can also be helpful for the concerned faculty members.

The FARSS can go through standards of OARS. You can also play vital role if you have any suggestions so that proper amendment can take place to improve the same for the benefit of entire research community.

As FARSS, you will be given a renowned, secure and free professional email address with 100 GB of space e.g. johnhall@globaljournals.org. This will include Webmail, Spam Assassin, Email Forwarders,Auto-Responders, Email Delivery Route tracing, etc.

The FARSS will be eligible for a free application of standardization of their researches. Standardization of research will be subject to acceptability within stipulated norms as the next step after publishing in a journal. We shall depute a team of specialized research professionals who will render their services for elevating your researches to next higher level, which is worldwide open standardization.

The FARSS member can apply for grading and certification of standards of their educational and Institutional Degrees to Open Association of Research, Society U.S.A.
Once you are designated as FARSS, you may send us a scanned copy of all of your credentials. OARS will verify, grade and certify them. This will be based on your academic records, quality of research papers published by you, and some more criteria. After certification of all your credentials by OARS, they will be published on your Fellow Profile link on website https://associationofresearch.org which will be helpful to upgrade the dignity.

The FARSS members can avail the benefits of free research podcasting in Global Research Radio with their research documents. After publishing the work, (including published elsewhere worldwide with proper authorization) you can upload your research paper with your recorded voice or you can utilize chargeable services of our professional RJs to record your paper in their voice on request.

The FARSS member also entitled to get the benefits of free research podcasting of their research documents through video clips. We can also streamline your conference videos and display your slides/ online slides and online research video clips at reasonable charges, on request.

The FARSS is eligible to earn from sales proceeds of his/her researches/reference/review Books or literature, while publishing with Global Journals. The FARSS can decide whether he/she would like to publish his/her research in a closed manner. In this case, whenever readers purchase that individual research paper for reading, maximum 60% of its profit earned as royalty by Global Journals, will be credited to his/her bank account. The entire entitled amount will be credited to his/her bank account exceeding limit of minimum fixed balance. There is no minimum time limit for collection. The FARSS member can decide its price and we can help in making the right decision.

The FARSS member is eligible to join as a paid peer reviewer at Global Journals Incorporation (USA) and can get remuneration of 15% of author fees, taken from the author of a respective paper. After reviewing 5 or more papers you can request to transfer the amount to your bank account.

MEMBER OF ASSOCIATION OF RESEARCH SOCIETY IN SCIENCE (MARSS)

The ' MARSS ' title is accorded to a selected professional after the approval of the Editor-in-Chief / Editorial Board Members/Dean.

The "MARSS" is a dignified ornament which is accorded to a person's name viz. Dr. John E. Hall, Ph.D., MARSS or William Walldroff, M.S., MARSS.

MARSS accrediting is an honor. It authenticates your research activities. After becoming MARSS, you can add 'MARSS' title with your name as you use this recognition as additional suffix to your status. This will definitely enhance and add more value and repute to your name. You may use it on your professional Counseling Materials such as CV, Resume, Visiting Card and Name Plate etc.

The following benefitscan be availed by you only for next three years from the date of certification.

MARSS designated members are entitled to avail a 25% discount while publishing their research papers (of a single author) in Global Journals Inc., if the same is accepted by our Editorial Board and Peer Reviewers. If you are a main author or co-author of a group of authors, you will get discount of 10%.

As MARSS, you will be given a renowned, secure and free professional email address with 30 GB of space e.g. johnhall@globaljournals.org. This will include Webmail, Spam Assassin, Email Forwarders,Auto-Responders, Email Delivery Route tracing, etc.

We shall provide you intimation regarding launching of e-version of journal of your stream time to time.This may be utilized in your library for the enrichment of knowledge of your students as well as it can also be helpful for the concerned faculty members.

The MARSS member can apply for approval, grading and certification of standards of their educational and Institutional Degrees to Open Association of Research, Society U.S.A.

Once you are designated as MARSS, you may send us a scanned copy of all of your credentials. OARS will verify, grade and certify them. This will be based on your academic records, quality of research papers published by you, and some more criteria.

It is mandatory to read all terms and conditions carefully.

AUXILIARY MEMBERSHIPS

Institutional Fellow of Global Journals Incorporation (USA)-OARS (USA)

Global Journals Incorporation (USA) is accredited by Open Association of Research Society, U.S.A (OARS) and in turn, affiliates research institutions as "Institutional Fellow of Open Association of Research Society" (IFOARS).

The "FARSC" is a dignified title which is accorded to a person's name viz. Dr. John E. Hall, Ph.D., FARSC or William Walldroff, M.S., FARSC.

The IFOARS institution is entitled to form a Board comprised of one Chairperson and three to five board members preferably from different streams. The Board will be recognized as "Institutional Board of Open Association of Research Society"-(IBOARS).

The Institute will be entitled to following benefits:

The IBOARS can initially review research papers of their institute and recommend them to publish with respective journal of Global Journals. It can also review the papers of other institutions after obtaining our consent. The second review will be done by peer reviewer of Global Journals Incorporation (USA) The Board is at liberty to appoint a peer reviewer with the approval of chairperson after consulting us.

The author fees of such paper may be waived off up to 40%.

The Global Journals Incorporation (USA) at its discretion can also refer double blind peer reviewed paper at their end to the board for the verification and to get recommendation for final stage of acceptance of publication.

The IBOARS can organize symposium/seminar/conference in their country on behalf of Global Journals Incorporation (USA)-OARS (USA). The terms and conditions can be discussed separately.

The Board can also play vital role by exploring and giving valuable suggestions regarding the Standards of "Open Association of Research Society, U.S.A (OARS)" so that proper amendment can take place for the benefit of entire research community. We shall provide details of particular standard only on receipt of request from the Board.

The board members can also join us as Individual Fellow with 40% discount on total fees applicable to Individual Fellow. They will be entitled to avail all the benefits as declared. Please visit Individual Fellow-sub menu of GlobalJournals.org to have more relevant details.

We shall provide you intimation regarding launching of e-version of journal of your stream time to time. This may be utilized in your library for the enrichment of knowledge of your students as well as it can also be helpful for the concerned faculty members.

After nomination of your institution as "Institutional Fellow" and constantly functioning successfully for one year, we can consider giving recognition to your institute to function as Regional/Zonal office on our behalf.
The board can also take up the additional allied activities for betterment after our consultation.

The following entitlements are applicable to individual Fellows:

Open Association of Research Society, U.S.A (OARS) By-laws states that an individual Fellow may use the designations as applicable, or the corresponding initials. The Credentials of individual Fellow and Associate designations signify that the individual has gained knowledge of the fundamental concepts. One is magnanimous and proficient in an expertise course covering the professional code of conduct, and follows recognized standards of practice.

Open Association of Research Society (US)/ Global Journals Incorporation (USA), as described in Corporate Statements, are educational, research publishing and professional membership organizations. Achieving our individual Fellow or Associate status is based mainly on meeting stated educational research requirements.

Disbursement of 40% Royalty earned through Global Journals : Researcher = 50%, Peer Reviewer = 37.50%, Institution = 12.50% E.g. Out of 40%, the 20% benefit should be passed on to researcher, 15 % benefit towards remuneration should be given to a reviewer and remaining 5% is to be retained by the institution.

We shall provide print version of 12 issues of any three journals [as per your requirement] out of our 38 journals worth $ 2376 USD.

Other:

The individual Fellow and Associate designations accredited by Open Association of Research Society (US) credentials signify guarantees following achievements:

- The professional accredited with Fellow honor, is entitled to various benefits viz. name, fame, honor, regular flow of income, secured bright future, social status etc.

- In addition to above, if one is single author, then entitled to 40% discount on publishing research paper and can get 10%discount if one is co-author or main author among group of authors.
- The Fellow can organize symposium/seminar/conference on behalf of Global Journals Incorporation (USA) and he/she can also attend the same organized by other institutes on behalf of Global Journals.
- The Fellow can become member of Editorial Board Member after completing 3yrs.
- The Fellow can earn 60% of sales proceeds from the sale of reference/review books/literature/publishing of research paper.
- Fellow can also join as paid peer reviewer and earn 15% remuneration of author charges and can also get an opportunity to join as member of the Editorial Board of Global Journals Incorporation (USA)
- • This individual has learned the basic methods of applying those concepts and techniques to common challenging situations. This individual has further demonstrated an in–depth understanding of the application of suitable techniques to a particular area of research practice.

Note :

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- In future, if the board feels the necessity to change any board member, the same can be done with the consent of the chairperson along with anyone board member without our approval.
- In case, the chairperson needs to be replaced then consent of 2/3rd board members are required and they are also required to jointly pass the resolution copy of which should be sent to us. In such case, it will be compulsory to obtain our approval before replacement.
- In case of "Difference of Opinion [if any]" among the Board members, our decision will be final and binding to everyone.

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Preferred Author Guidelines

We accept the manuscript submissions in any standard (generic) format.

We typeset manuscripts using advanced typesetting tools like Adobe In Design, CorelDraw, TeXnicCenter, and TeXStudio. We usually recommend authors submit their research using any standard format they are comfortable with, and let Global Journals do the rest.

Alternatively, you can download our basic template from https://globaljournals.org/Template.zip

Authors should submit their complete paper/article, including text illustrations, graphics, conclusions, artwork, and tables. Authors who are not able to submit manuscript using the form above can email the manuscript department at submit@globaljournals.org or get in touch with chiefeditor@globaljournals.org if they wish to send the abstract before submission.

Before and during Submission

Authors must ensure the information provided during the submission of a paper is authentic. Please go through the following checklist before submitting:

1. Authors must go through the complete author guideline and understand and *agree to Global Journals' ethics and code of conduct,* along with author responsibilities.
2. Authors must accept the privacy policy, terms, and conditions of Global Journals.
3. Ensure corresponding author's email address and postal address are accurate and reachable.
4. Manuscript to be submitted must include keywords, an abstract, a paper title, co-author(s') names and details (email address, name, phone number, and institution), figures and illustrations in vector format including appropriate captions, tables, including titles and footnotes, a conclusion, results, acknowledgments and references.
5. Authors should submit paper in a ZIP archive if any supplementary files are required along with the paper.
6. Proper permissions must be acquired for the use of any copyrighted material.
7. Manuscript submitted *must not have been submitted or published elsewhere* and all authors must be aware of the submission.

Declaration of Conflicts of Interest

It is required for authors to declare all financial, institutional, and personal relationships with other individuals and organizations that could influence (bias) their research.

Policy on Plagiarism

Plagiarism is not acceptable in Global Journals submissions at all.

Plagiarized content will not be considered for publication. We reserve the right to inform authors' institutions about plagiarism detected either before or after publication. If plagiarism is identified, we will follow COPE guidelines:

Authors are solely responsible for all the plagiarism that is found. The author must not fabricate, falsify or plagiarize existing research data. The following, if copied, will be considered plagiarism:

- Words (language)
- Ideas
- Findings
- Writings
- Diagrams
- Graphs
- Illustrations
- Lectures

- Printed material
- Graphic representations
- Computer programs
- Electronic material
- Any other original work

Authorship Policies

Global Journals follows the definition of authorship set up by the Open Association of Research Society, USA. According to its guidelines, authorship criteria must be based on:

1. Substantial contributions to the conception and acquisition of data, analysis, and interpretation of findings.
2. Drafting the paper and revising it critically regarding important academic content.
3. Final approval of the version of the paper to be published.

Changes in Authorship

The corresponding author should mention the name and complete details of all co-authors during submission and in manuscript. We support addition, rearrangement, manipulation, and deletions in authors list till the early view publication of the journal. We expect that corresponding author will notify all co-authors of submission. We follow COPE guidelines for changes in authorship.

Copyright

During submission of the manuscript, the author is confirming an exclusive license agreement with Global Journals which gives Global Journals the authority to reproduce, reuse, and republish authors' research. We also believe in flexible copyright terms where copyright may remain with authors/employers/institutions as well. Contact your editor after acceptance to choose your copyright policy. You may follow this form for copyright transfers.

Appealing Decisions

Unless specified in the notification, the Editorial Board's decision on publication of the paper is final and cannot be appealed before making the major change in the manuscript.

Acknowledgments

Contributors to the research other than authors credited should be mentioned in Acknowledgments. The source of funding for the research can be included. Suppliers of resources may be mentioned along with their addresses.

Declaration of funding sources

Global Journals is in partnership with various universities, laboratories, and other institutions worldwide in the research domain. Authors are requested to disclose their source of funding during every stage of their research, such as making analysis, performing laboratory operations, computing data, and using institutional resources, from writing an article to its submission. This will also help authors to get reimbursements by requesting an open access publication letter from Global Journals and submitting to the respective funding source.

Preparing your Manuscript

Authors can submit papers and articles in an acceptable file format: MS Word (doc, docx), LaTeX (.tex, .zip or .rar including all of your files), Adobe PDF (.pdf), rich text format (.rtf), simple text document (.txt), Open Document Text (.odt), and Apple Pages (.pages). Our professional layout editors will format the entire paper according to our official guidelines. This is one of the highlights of publishing with Global Journals—authors should not be concerned about the formatting of their paper. Global Journals accepts articles and manuscripts in every major language, be it Spanish, Chinese, Japanese, Portuguese, Russian, French, German, Dutch, Italian, Greek, or any other national language, but the title, subtitle, and abstract should be in English. This will facilitate indexing and the pre-peer review process.

The following is the official style and template developed for publication of a research paper. Authors are not required to follow this style during the submission of the paper. It is just for reference purposes.

Manuscript Style Instruction (Optional)

- Microsoft Word Document Setting Instructions.
- Font type of all text should be Swis721 Lt BT.
- Page size: 8.27" x 11'", left margin: 0.65, right margin: 0.65, bottom margin: 0.75.
- Paper title should be in one column of font size 24.
- Author name in font size of 11 in one column.
- Abstract: font size 9 with the word "Abstract" in bold italics.
- Main text: font size 10 with two justified columns.
- Two columns with equal column width of 3.38 and spacing of 0.2.
- First character must be three lines drop-capped.
- The paragraph before spacing of 1 pt and after of 0 pt.
- Line spacing of 1 pt.
- Large images must be in one column.
- The names of first main headings (Heading 1) must be in Roman font, capital letters, and font size of 10.
- The names of second main headings (Heading 2) must not include numbers and must be in italics with a font size of 10.

Structure and Format of Manuscript

The recommended size of an original research paper is under 15,000 words and review papers under 7,000 words. Research articles should be less than 10,000 words. Research papers are usually longer than review papers. Review papers are reports of significant research (typically less than 7,000 words, including tables, figures, and references)

A research paper must include:

a) A title which should be relevant to the theme of the paper.
b) A summary, known as an abstract (less than 150 words), containing the major results and conclusions.
c) Up to 10 keywords that precisely identify the paper's subject, purpose, and focus.
d) An introduction, giving fundamental background objectives.
e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition, sources of information must be given, and numerical methods must be specified by reference.
f) Results which should be presented concisely by well-designed tables and figures.
g) Suitable statistical data should also be given.
h) All data must have been gathered with attention to numerical detail in the planning stage.

Design has been recognized to be essential to experiments for a considerable time, and the editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned unrefereed.

i) Discussion should cover implications and consequences and not just recapitulate the results; conclusions should also be summarized.
j) There should be brief acknowledgments.
k) There ought to be references in the conventional format. Global Journals recommends APA format.

Authors should carefully consider the preparation of papers to ensure that they communicate effectively. Papers are much more likely to be accepted if they are carefully designed and laid out, contain few or no errors, are summarizing, and follow instructions. They will also be published with much fewer delays than those that require much technical and editorial correction.

The Editorial Board reserves the right to make literary corrections and suggestions to improve brevity.

Format Structure

It is necessary that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

All manuscripts submitted to Global Journals should include:

Title

The title page must carry an informative title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) where the work was carried out.

Author details

The full postal address of any related author(s) must be specified.

Abstract

The abstract is the foundation of the research paper. It should be clear and concise and must contain the objective of the paper and inferences drawn. It is advised to not include big mathematical equations or complicated jargon.

Many researchers searching for information online will use search engines such as Google, Yahoo or others. By optimizing your paper for search engines, you will amplify the chance of someone finding it. In turn, this will make it more likely to be viewed and cited in further works. Global Journals has compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Keywords

A major lynchpin of research work for the writing of research papers is the keyword search, which one will employ to find both library and internet resources. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining, and indexing.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy: planning of a list of possible keywords and phrases to try.

Choice of the main keywords is the first tool of writing a research paper. Research paper writing is an art. Keyword search should be as strategic as possible.

One should start brainstorming lists of potential keywords before even beginning searching. Think about the most important concepts related to research work. Ask, “What words would a source have to include to be truly valuable in a research paper?” Then consider synonyms for the important words.

It may take the discovery of only one important paper to steer in the right keyword direction because, in most databases, the keywords under which a research paper is abstracted are listed with the paper.

Numerical Methods

Numerical methods used should be transparent and, where appropriate, supported by references.

Abbreviations

Authors must list all the abbreviations used in the paper at the end of the paper or in a separate table before using them.

Formulas and equations

Authors are advised to submit any mathematical equation using either MathJax, KaTeX, or LaTeX, or in a very high-quality image.

Tables, Figures, and Figure Legends

Tables: Tables should be cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g., Table 4, a self-explanatory caption, and be on a separate sheet. Authors must submit tables in an editable format and not as images. References to these tables (if any) must be mentioned accurately.

Figures

Figures are supposed to be submitted as separate files. Always include a citation in the text for each figure using Arabic numbers, e.g., Fig. 4. Artwork must be submitted online in vector electronic form or by emailing it.

Preparation of Eletronic Figures for Publication

Although low-quality images are sufficient for review purposes, print publication requires high-quality images to prevent the final product being blurred or fuzzy. Submit (possibly by e-mail) EPS (line art) or TIFF (halftone/ photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Avoid using pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings). Please give the data for figures in black and white or submit a Color Work Agreement form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution at final image size ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs): >350 dpi; figures containing both halftone and line images: >650 dpi.

Color charges: Authors are advised to pay the full cost for the reproduction of their color artwork. Hence, please note that if there is color artwork in your manuscript when it is accepted for publication, we would require you to complete and return a Color Work Agreement form before your paper can be published. Also, you can email your editor to remove the color fee after acceptance of the paper.

Tips for Writing a Good Quality Science Frontier Research Paper

Techniques for writing a good quality Science Frontier Research paper:

1. Choosing the topic: In most cases, the topic is selected by the interests of the author, but it can also be suggested by the guides. You can have several topics, and then judge which you are most comfortable with. This may be done by asking several questions of yourself, like "Will I be able to carry out a search in this area? Will I find all necessary resources to accomplish the search? Will I be able to find all information in this field area?" If the answer to this type of question is "yes," then you ought to choose that topic. In most cases, you may have to conduct surveys and visit several places. Also, you might have to do a lot of work to find all the rises and falls of the various data on that subject. Sometimes, detailed information plays a vital role, instead of short information. Evaluators are human: The first thing to remember is that evaluators are also human beings. They are not only meant for rejecting a paper. They are here to evaluate your paper. So present your best aspect.

2. Think like evaluators: If you are in confusion or getting demotivated because your paper may not be accepted by the evaluators, then think, and try to evaluate your paper like an evaluator. Try to understand what an evaluator wants in your research paper, and you will automatically have your answer. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

3. Ask your guides: If you are having any difficulty with your research, then do not hesitate to share your difficulty with your guide (if you have one). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work, then ask your supervisor to help you with an alternative. He or she might also provide you with a list of essential readings.

4. Use of computer is recommended: As you are doing research in the field of science frontier then this point is quite obvious. Use right software: Always use good quality software packages. If you are not capable of judging good software, then you can lose the quality of your paper unknowingly. There are various programs available to help you which you can get through the internet.

5. Use the internet for help: An excellent start for your paper is using Google. It is a wondrous search engine, where you can have your doubts resolved. You may also read some answers for the frequent question of how to write your research paper or find a model research paper. You can download books from the internet. If you have all the required books, place importance on reading, selecting, and analyzing the specified information. Then sketch out your research paper. Use big pictures: You may use encyclopedias like Wikipedia to get pictures with the best resolution. At Global Journals, you should strictly follow here.

6. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right? It is a good habit which helps to not lose your continuity. You should always use bookmarks while searching on the internet also, which will make your search easier.

7. Revise what you wrote: When you write anything, always read it, summarize it, and then finalize it.

8. Make every effort: Make every effort to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in the introduction—what is the need for a particular research paper. Polish your work with good writing skills and always give an evaluator what he wants. Make backups: When you are going to do any important thing like making a research paper, you should always have backup copies of it either on your computer or on paper. This protects you from losing any portion of your important data.

9. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several unnecessary diagrams will degrade the quality of your paper by creating a hodgepodge. So always try to include diagrams which were made by you to improve the readability of your paper. Use of direct quotes: When you do research relevant to literature, history, or current affairs, then use of quotes becomes essential, but if the study is relevant to science, use of quotes is not preferable.

10. Use proper verb tense: Use proper verb tenses in your paper. Use past tense to present those events that have happened. Use present tense to indicate events that are going on. Use future tense to indicate events that will happen in the future. Use of wrong tenses will confuse the evaluator. Avoid sentences that are incomplete.

11. Pick a good study spot: Always try to pick a spot for your research which is quiet. Not every spot is good for studying.

12. Know what you know: Always try to know what you know by making objectives, otherwise you will be confused and unable to achieve your target.

13. Use good grammar: Always use good grammar and words that will have a positive impact on the evaluator; use of good vocabulary does not mean using tough words which the evaluator has to find in a dictionary. Do not fragment sentences. Eliminate one-word sentences. Do not ever use a big word when a smaller one would suffice.

Verbs have to be in agreement with their subjects. In a research paper, do not start sentences with conjunctions or finish them with prepositions. When writing formally, it is advisable to never split an infinitive because someone will (wrongly) complain. Avoid clichés like a disease. Always shun irritating alliteration. Use language which is simple and straightforward. Put together a neat summary.

14. Arrangement of information: Each section of the main body should start with an opening sentence, and there should be a changeover at the end of the section. Give only valid and powerful arguments for your topic. You may also maintain your arguments with records.

15. Never start at the last minute: Always allow enough time for research work. Leaving everything to the last minute will degrade your paper and spoil your work.

16. Multitasking in research is not good: Doing several things at the same time is a bad habit in the case of research activity. Research is an area where everything has a particular time slot. Divide your research work into parts, and do a particular part in a particular time slot.

17. Never copy others' work: Never copy others' work and give it your name because if the evaluator has seen it anywhere, you will be in trouble. Take proper rest and food: No matter how many hours you spend on your research activity, if you are not taking care of your health, then all your efforts will have been in vain. For quality research, take proper rest and food.

18. Go to seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

19. Refresh your mind after intervals: Try to give your mind a rest by listening to soft music or sleeping in intervals. This will also improve your memory. Acquire colleagues: Always try to acquire colleagues. No matter how sharp you are, if you acquire colleagues, they can give you ideas which will be helpful to your research.

20. Think technically: Always think technically. If anything happens, search for its reasons, benefits, and demerits. Think and then print: When you go to print your paper, check that tables are not split, headings are not detached from their descriptions, and page sequence is maintained.

21. Adding unnecessary information: Do not add unnecessary information like "I have used MS Excel to draw graphs." Irrelevant and inappropriate material is superfluous. Foreign terminology and phrases are not apropos. One should never take a broad view. Analogy is like feathers on a snake. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Never oversimplify: When adding material to your research paper, never go for oversimplification; this will definitely irritate the evaluator. Be specific. Never use rhythmic redundancies. Contractions shouldn't be used in a research paper. Comparisons are as terrible as clichés. Give up ampersands, abbreviations, and so on. Remove commas that are not necessary. Parenthetical words should be between brackets or commas. Understatement is always the best way to put forward earth-shaking thoughts. Give a detailed literary review.

22. Report concluded results: Use concluded results. From raw data, filter the results, and then conclude your studies based on measurements and observations taken. An appropriate number of decimal places should be used. Parenthetical remarks are prohibited here. Proofread carefully at the final stage. At the end, give an outline to your arguments. Spot perspectives of further study of the subject. Justify your conclusion at the bottom sufficiently, which will probably include examples.

23. Upon conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print for the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects of your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form which is presented in the guidelines using the template.
- Please note the criteria peer reviewers will use for grading the final paper.

Final points:

One purpose of organizing a research paper is to let people interpret your efforts selectively. The journal requires the following sections, submitted in the order listed, with each section starting on a new page:

The introduction: This will be compiled from reference matter and reflect the design processes or outline of basis that directed you to make a study. As you carry out the process of study, the method and process section will be constructed like that. The results segment will show related statistics in nearly sequential order and direct reviewers to similar intellectual paths throughout the data that you gathered to carry out your study.

The discussion section:

This will provide understanding of the data and projections as to the implications of the results. The use of good quality references throughout the paper will give the effort trustworthiness by representing an alertness to prior workings.

Writing a research paper is not an easy job, no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record-keeping are the only means to make straightforward progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear: Adhere to recommended page limits.

Mistakes to avoid:

- Insertion of a title at the foot of a page with subsequent text on the next page.
- Separating a table, chart, or figure—confine each to a single page.
- Submitting a manuscript with pages out of sequence.
- In every section of your document, use standard writing style, including articles ("a" and "the").
- Keep paying attention to the topic of the paper.
- Use paragraphs to split each significant point (excluding the abstract).
- Align the primary line of each section.
- Present your points in sound order.
- Use present tense to report well-accepted matters.
- Use past tense to describe specific results.
- Do not use familiar wording; don't address the reviewer directly. Don't use slang or superlatives.
- Avoid use of extra pictures—include only those figures essential to presenting results.

Title page:

Choose a revealing title. It should be short and include the name(s) and address(es) of all authors. It should not have acronyms or abbreviations or exceed two printed lines.

Abstract: This summary should be two hundred words or less. It should clearly and briefly explain the key findings reported in the manuscript and must have precise statistics. It should not have acronyms or abbreviations. It should be logical in itself. Do not cite references at this point.

An abstract is a brief, distinct paragraph summary of finished work or work in development. In a minute or less, a reviewer can be taught the foundation behind the study, common approaches to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Use comprehensive sentences, and do not sacrifice readability for brevity; you can maintain it succinctly by phrasing sentences so that they provide more than a lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study with the subsequent elements in any summary. Try to limit the initial two items to no more than one line each.

Reason for writing the article—theory, overall issue, purpose.

- Fundamental goal.
- To-the-point depiction of the research.
- Consequences, including definite statistics—if the consequences are quantitative in nature, account for this; results of any numerical analysis should be reported. Significant conclusions or questions that emerge from the research.

Approach:

- Single section and succinct.
- An outline of the job done is always written in past tense.
- Concentrate on shortening results—limit background information to a verdict or two.
- Exact spelling, clarity of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else.

Introduction:

The introduction should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable of comprehending and calculating the purpose of your study without having to refer to other works. The basis for the study should be offered. Give the most important references, but avoid making a comprehensive appraisal of the topic. Describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will give no attention to your results. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here.

The following approach can create a valuable beginning:

- Explain the value (significance) of the study.
- Defend the model—why did you employ this particular system or method? What is its compensation? Remark upon its appropriateness from an abstract point of view as well as pointing out sensible reasons for using it.
- Present a justification. State your particular theory(-ies) or aim(s), and describe the logic that led you to choose them.
- Briefly explain the study's tentative purpose and how it meets the declared objectives.

Approach:

Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done. Sort out your thoughts; manufacture one key point for every section. If you make the four points listed above, you will need at least four paragraphs. Present surrounding information only when it is necessary to support a situation. The reviewer does not desire to read everything you know about a topic. Shape the theory specifically—do not take a broad view.

As always, give awareness to spelling, simplicity, and correctness of sentences and phrases.

Procedures (methods and materials):

This part is supposed to be the easiest to carve if you have good skills. A soundly written procedures segment allows a capable scientist to replicate your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order, but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt to give the least amount of information that would permit another capable scientist to replicate your outcome, but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section.

When a technique is used that has been well-described in another section, mention the specific item describing the way, but draw the basic principle while stating the situation. The purpose is to show all particular resources and broad procedures so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step-by-step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

Materials may be reported in part of a section or else they may be recognized along with your measures.

Methods:

- Report the method and not the particulars of each process that engaged the same methodology.
- Describe the method entirely.
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures.
- Simplify—detail how procedures were completed, not how they were performed on a particular day.
- If well-known procedures were used, account for the procedure by name, possibly with a reference, and that's all.

Approach:

It is embarrassing to use vigorous voice when documenting methods without using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result, when writing up the methods, most authors use third person passive voice.

Use standard style in this and every other part of the paper—avoid familiar lists, and use full sentences.

What to keep away from:

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings—save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part as entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Use statistics and tables, if suitable, to present consequences most efficiently.

You must clearly differentiate material which would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matters should not be submitted at all except if requested by the instructor.

Content:

- Sum up your conclusions in text and demonstrate them, if suitable, with figures and tables.
- In the manuscript, explain each of your consequences, and point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation of an exacting study.
- Explain results of control experiments and give remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or manuscript.

What to stay away from:

- Do not discuss or infer your outcome, report surrounding information, or try to explain anything.
- Do not include raw data or intermediate calculations in a research manuscript.
- Do not present similar data more than once.
- A manuscript should complement any figures or tables, not duplicate information.
- Never confuse figures with tables—there is a difference.

Approach:

As always, use past tense when you submit your results, and put the whole thing in a reasonable order.

Put figures and tables, appropriately numbered, in order at the end of the report.

If you desire, you may place your figures and tables properly within the text of your results section.

Figures and tables:

If you put figures and tables at the end of some details, make certain that they are visibly distinguished from any attached appendix materials, such as raw facts. Whatever the position, each table must be titled, numbered one after the other, and include a heading. All figures and tables must be divided from the text.

Discussion:

The discussion is expected to be the trickiest segment to write. A lot of papers submitted to the journal are discarded based on problems with the discussion. There is no rule for how long an argument should be.

Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implications of the study. The purpose here is to offer an understanding of your results and support all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of results should be fully described.

Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact, you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved the prospect, and let it drop at that. Make a decision as to whether each premise is supported or discarded or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."

Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work.

- You may propose future guidelines, such as how an experiment might be personalized to accomplish a new idea.
- Give details of all of your remarks as much as possible, focusing on mechanisms.
- Make a decision as to whether the tentative design sufficiently addressed the theory and whether or not it was correctly restricted. Try to present substitute explanations if they are sensible alternatives.
- One piece of research will not counter an overall question, so maintain the large picture in mind. Where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

When you refer to information, differentiate data generated by your own studies from other available information. Present work done by specific persons (including you) in past tense.

Describe generally acknowledged facts and main beliefs in present tense.

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Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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Global Journal of Science Frontier Research

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ISSN 9755896