Dynamic Scalar Potential and the Electrokinetic Electric Field

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Citation

Abstract
In the article is described the new physical phenomenon, which indicates that in the superconductors the stationary electric field can exist. This field exists in the surface layer of superconductor and it is directed normal to its surface. Since the field indicated is the consequence of the kinetic motion of charges in the superconductor, it can be named electrokinetic electric field. The value of this field is small, but it are new, previously unknown, property of the superconductive state.

1. Introduction
The electrodynamics of superconductors does not assume the presence in them of stationary electrical fields on, since this would lead to an infinite increase in the speed of current carriers. In the article is described earlier not the known physical phenomenon, which is assumed the presence of stationary electrical fields on in the superconductors. These fields are the consequence of the kinetic motion of charges and therefore they can be named kinetic electric fields. The value of this field is small, but it are new, previously unknown, property of the superconductive state.

2. Electrodynamics of Plasmo-Like Media
By plasma media we will understand such, in which the charges can move without the losses. To such media in the first approximation, can be related the superconductors, free electrons or ions in the vacuum. In the media indicated the equation of motion of electron takes the form:

\[ m \frac{d\vec{v}}{dt} = e\vec{E}, \]

(2.1)

where \( m \) is mass electron, \( e \) is the electron charge, \( \vec{E} \) is the tension of electric field, \( \vec{v} \) is speed of the motion of charge.

Using an expression for the current density

\[ j = ne\vec{v}, \]

(2.2)

from equation (2.1) we obtain the current density of the conductivity

\[ J_l = \frac{ne^2}{m} \int \vec{E} \, dt. \]

(2.3)

In relationship (2.2) and (2.3) the value \( n \) represents electron density. After introducing the designation
we find

\[ j_k = \frac{1}{L_k} \int \dot{E} \, dt . \]  

(2.5)

In this case the value \( L_k \) presents the specific kinetic inductance of charge carriers [2-6]. Its existence connected with the fact that charge, having a mass, possesses inertia properties.

\[ j_k = -\frac{1}{\alpha L_k} E_0 \cos \omega t . \]  

(2.6)

For the mathematical description of electrodynamic processes the trigonometric functions will be here and throughout, instead of the complex quantities, used so that would be well visible the phase relationships between the vectors, which represent electric fields and current densities.

From relationship (2.5) and (2.6) is evident that \( j_k \) presents inductive current, since its phase is late with respect to the tension of electric field to the angle \( \frac{\pi}{2} \).

During the presence of summed current it is necessary to consider bias current

\[ j_k = E_0 \frac{\partial}{\partial t} + \frac{1}{L_k} \int \dot{E} \, dt . \]

Maxwell’s equations for this case take the form:

\[ \text{rot} \: \dot{E} = -\mu_0 \frac{\partial \dot{H}}{\partial t} , \]  

\[ \text{rot} \: \dot{H} = \varepsilon_0 \frac{\partial \dot{E}}{\partial t} + \frac{1}{L_k} \int \dot{E} \, dt , \]  

(2.7)

where \( \varepsilon_0 \) and \( \mu_0 \) are dielectric and magnetic constant of vacuum.

System of equations (2.7) describes the properties of superconductors. From it we obtain

\[ \text{rot} \: \dot{H} + \mu_0 E_0 \frac{\partial^2 \dot{H}}{\partial t^2} + \frac{\mu_0}{L_k} \dot{H} = 0 . \]  

(2.8)

For the case fields on, time-independent, equation (2.8) passes into the London’s equation

\[ \text{rot} \: \dot{H} + \frac{\mu_0}{L_k} \dot{H} = 0 , \]

where \( \lambda^2 = \frac{L_k}{\mu_0} \) is London’s depth of penetration.

Fields on wave equation in this case it appears as follows for the electrical:

\[ \text{rot} \: \dot{E} + \mu_0 \varepsilon_0 \frac{\partial^2 \dot{E}}{\partial t^2} + \frac{\mu_0}{L_k} \dot{E} = 0 . \]

For constant electrical fields on it is possible to write down

\[ \text{rot} \: \dot{E} + \frac{\mu_0}{L_k} \dot{E} = 0 . \]

Consequently, dc fields penetrate the superconductor in the same manner as for magnetic, diminishing exponentially. However, the density of current in this case grows according to the linear law

\[ j_k = \frac{1}{L_k} \int \dot{E} \, dt . \]

This means that stationary electric fields in the superconductor it can exist only until current density reaches its critical value for this type of superconductor.

If around the point in question is some static configuration of charges, then the tension of electric field will be at the particular point determined by the relationship of , where the scalar potential at the assigned point, determined by the assigned configuration of charges. If we change the arrangement of charges, then this new configuration will correspond other values of scalar potential, and, therefore, also other values of the tension of electric field.

In the electrodynamics the fundamental law of induction is Faraday law. It is written as follows:

\[ \oint \dot{E} \, dl = -\frac{\partial \Phi_b}{\partial t} = -\mu \oint \frac{\partial \dot{H}}{\partial t} \, ds = -\oint \frac{\partial \dot{B}}{\partial t} \, ds \]  

(2.9)

where \( B = \mu \dot{H} \) is magnetic induction vector, \( \Phi_b = \mu \oint \dot{H} \, ds \) is flow of magnetic induction, and \( \mu = \varepsilon_0 \mu_0 \) is magnetic permeability of medium. It follows from this law that the circulation integral of the vector of electric field is equal to a change in the flow of magnetic induction through the area, which this outline covers. It is immediately necessary to emphasize the circumstance that the law in question presents the processes of mutual induction, since. For obtaining the circulation integral of the vector \( \dot{E} \) we take the strange magnetic field, formed by strange source. From relationship (2.9) obtain the first Maxwell’s equation.
rot \vec{E} = -\frac{\partial \vec{B}}{\partial t}. \hspace{1cm} (2.10)

Let us immediately point out to the terminological error. Faraday law should be called not the law of electromagnetic, as is customary in the existing literature, but by the law of magnetoelectric induction, since, a change in the magnetic fields on it leads to the appearance of electrical fields on, but not vice versa.

Let us assume that in the region of the arrangement of the outline of integration there is a certain local vector \( \vec{A}_H \), which satisfies the equality
\[
\mu \oint \vec{A}_H \cdot d\vec{l} = \Phi_H,
\]
where the outline of the integration coincides with the outline of integration in relationship (2.9), and the vector of is determined in all sections of this outline, then
\[
\vec{E} = -\mu \frac{\partial \vec{A}_H}{\partial t}. \hspace{1cm} (2.11)
\]

Introduced thus vector \( \vec{A}_H \) determines the local connection between it and by electric field. It is not difficult to show that introduced thus vector \( \vec{A}_H \), is connected with the magnetic field with the following relationship:
\[
rot \vec{A}_H = \vec{H}. \hspace{1cm} (2.12)
\]

At those points of the space, where
\[
rot \vec{A}_H = 0,
\]
magnetic field is absent.

Thus, we will consider that the vector \( \vec{H} \) exists by a consequence of the presence of the vector \( \vec{A}_H \), but not vice versa.

If there is a straight conductor with the current, then around it also there is a field of vector potential, the truth in this case \( \text{rot} \vec{A}_H \neq 0 \) in the environments of this conductor is, therefore, located also the magnetic field, which changes with a change of the current in the conductor. The section of wire by the length \( dl \), over which flows the current \( I \), generates in the distant zone (it is thought that the distance \( r \) considerably more than the length of section) the vector potential
\[
d\vec{A}_H(r) = \frac{Idl}{4\pi r}.
\]

Until now, resolution of a question about the appearance of electrical fields on in different inertial moving systems (IS) it was possible to achieve in two ways. The first - consisted in the calculation of the Lorentz force, which acts on the moving charges, the alternate path consisted in the measurement of a change in the magnetic flux through the outline being investigated. Both methods gave identical result, and this was incomprehensible [10]. In connection with the incomprehension of physical nature of this state of affairs they began to consider that the unipolar generator is an exception to the rule of flow [10]. Let us examine this situation in more detail.

In order to answer the presented question, should be somewhat changed relationship (2.3), after replacing in it partial derivative by the complete:
\[
\vec{E}' = -\mu \frac{d\vec{A}_H}{dt} \hspace{1cm} (2.13)
\]

Prime near the vector \( \vec{E} \) means that this field is determined in the moving coordinate system, while the vector \( \vec{A}_H \) it is determined in the fixed system. This means that the vector potential can have not only local, but also convection derivative, i.e., it can change both due to the change in the time and due to the motion in the three-dimensional changing field of this potential. In this case relationship (2.16) can be rewritten as follows:
\[
\vec{E}' = -\mu \frac{d\vec{A}_H}{dt} - \mu (\vec{v} \nabla) \vec{A}_H,
\]
where \( \vec{v} \) is speed of the prime system. If vector potential on time does not depend, we obtain
\[
\vec{F}_{\text{vH}}' = -\mu e (\vec{v} \nabla) \vec{A}_H.
\]

This force depends only on the gradients of vector potential and charge rate.

The charge, which moves in the field of the vector potential \( \vec{A}_H \) with the speed \( \vec{v} \), possesses potential energy [10]
\[
W = -e\mu \vec{A}_H. \hspace{1cm} (2.14)
\]

Therefore must exist one additional force, which acts on the charge in the moving coordinate system, namely:
\[
\vec{F}_{\text{vH}}' = -\text{grad} W = e\mu \text{grad} (\vec{v} \vec{A}_H) \hspace{1cm} (2.14)
\]

Consequently, the value \( e\mu (\vec{v} \vec{A}_H) \) plays the same role as the scalar potential \( \phi \), whose gradient gives the force, which acts on the charge. Consequently, the composite force, which acts on the charge, which moves in the field of vector potential, can have three components and will be written down as
\[
\vec{F}' = -e\mu \frac{d\vec{A}_H}{dt} - e\mu (\vec{v} \nabla) \vec{A}_H + e\mu \text{grad} (\vec{v} \vec{A}_H). \hspace{1cm} (2.14)
\]

The first of the components of this force acts on the fixed charge, when vector potential changes in the time and has local time derivative. Second component is connected with
the motion of charge in the three-dimensional changing field of this potential. Entirely different nature in force, which is determined by last term of relationship (2.14). It is connected with the fact that the charge, which moves in the field of vector potential, it possesses potential energy, whose gradient gives force. From relationship (2.14) follows

$$\dot{E}' = -\mu \frac{\partial \tilde{A}_i}{\partial t} - \mu (\tilde{v}\nabla) \tilde{A}_i + \mu \text{grad} (\tilde{v} \tilde{A}_i).$$  \hspace{1cm} (2.15)

This is a complete law of mutual induction. It defines all electric fields, which can appear at the assigned point of space, this point can be both the fixed and that moving. This united law includes and Faraday law and that part of the Lorentz force, which is connected with the motion of charge in the magnetic field, and without any exceptions gives answer to all questions, which are concerned mutual magnetoelectric induction. It is significant, that, if we take rotor from both parts of equality (2.15), attempting to obtain the first Maxwell's equation, then it will be immediately lost the essential part of the information, since rotor from the gradient is identically equal to zero.

If we isolate those forces, which are connected with the motion of charge in the three-dimensional changing field of vector potential, and to consider that

$$\mu \text{grad} (\tilde{v} \tilde{A}_i) = -\mu (\tilde{v}\nabla) \tilde{A}_i = \mu [\tilde{v} \times \text{rot} \tilde{A}_i],$$

that from (2.14) we will obtain

$$\dot{F}' = e\mu [\tilde{v} \times \text{rot} \tilde{A}_i]$$ \hspace{1cm} (2.16)

and, taking into account (2.12), let us write down

$$\dot{F}' = e\mu [\tilde{v} \times \tilde{H}],$$ \hspace{1cm} (2.17)

or

$$\dot{E}' = \mu [\tilde{v} \times \tilde{H}],$$ \hspace{1cm} (2.18)

and it is final

$$\dot{F}' = e\tilde{E} + e\dot{E}' = -e \frac{\partial \tilde{A}_i}{\partial t} + e\mu [\tilde{v} \times \tilde{H}].$$ \hspace{1cm} (2.19)

Can see that relationship (2.19) presents Lorentz force, however, this not thus. In this relationship the field $\tilde{E}$ and the field $\dot{E}'$ are induction. The first equation is connected with a change of the vector potential with time, the second is obliged to the motion of charge in the three-dimensional changing field of this potential. In order to obtain the total force, which acts on the charge, necessary to the right side of relationship (2.19) to add the term $-e \text{grad} \phi$

$$\dot{F}' = -e \text{grad} \phi + e\tilde{E} + e\mu [\tilde{v} \times \tilde{H}],$$

where $\phi$ is scalar potential at the observation point. In this case relationship (2.15) can be rewritten as follows:

$$\dot{E}' = -\mu \frac{\partial \tilde{A}_i}{\partial t} - \mu (\tilde{v}\nabla) \tilde{A}_i + \mu \text{grad} (\tilde{v} \tilde{A}_i) - \text{grad} \phi.$$ \hspace{1cm} (2.20)

or, after writing down the first two members of the right side of relationship (2.20) as the derivative of vector potential on the time, and also, after introducing under the sign of gradient two last terms, we will obtain

$$\dot{E}' = -\mu \frac{\partial \tilde{A}_i}{\partial t} + \text{grad} (\tilde{v} \dot{A}_i) - \phi.$$ \hspace{1cm} (2.21)

If both parts of relationship (2.20) are multiplied by the magnitude of the charge, then will come out the total force, which acts on the charge. From Lorentz force it will differ in terms of the force $-e\mu \frac{\partial \tilde{A}_i}{\partial t}$. From relationship (2.21) it is evident that the value $\mu (\tilde{v} \dot{A}_i) - \phi$ plays the role of the generalized scalar potential.

The Maxwell's second equation in the terms of vector potential can be written down as follows:

$$\text{rot rot} \tilde{A}_i = \tilde{j} (\dot{A}_i).$$ \hspace{1cm} (2.22)

where $\dot{j} (\dot{A}_i)$ is the current density, which presents functional from the vector potential.

In the superconductor the current density is determined by the relationship

$$\dot{j} (\dot{A}_i) = -\frac{\mu}{L_k} \dot{A}_i,$$ \hspace{1cm} (2.23)

where $L_k = \frac{m}{ne^2}$ is kinetic inductance of charges.

The dependence of current density on the coordinate in the superconductor is determined by relationship [1]

$$\dot{j} (z) = \dot{j}_0 e^{-\frac{z}{L_k}},$$ \hspace{1cm} (2.24)

where $z$ is coordinate, directed into the depths of the superconductor, $\dot{j}_0$ is current density on the surface of superconductor.

Using relationship (2.2) we obtain the dependence of the electron velocity in the superconductor on the coordinate

$$\tilde{v} (z) = \tilde{v}_0 e^{-\frac{z}{L_k}}.$$ \hspace{1cm} (2.25)

Using relationships (2.23-2.25), from relationship (2.21) we obtain the value of kinetic electric field with the presence of the steady currents

$$E_k = -\mu \langle \text{grad} (\tilde{v} \dot{A}_i) \rangle = -2\mu_0 ne\lambda_s \tilde{v}_0^2 \tilde{v} e^{-\frac{2z}{L_k}}.$$ \hspace{1cm} (2.26)
Consequently, when, in the surface layer, the superconductor of steady currents is present, in this layer exists the electric field, normal to the surface, dependence which from coordinate is determined by relationship (2.26). This field can be named electrokinetic, since it is generated by the kinetic electron motion.

The magnetic field on its surface of superconductor, equal to specific current, can be determined from the relationship

\[ H_0 = nev_0 \lambda_z \]

Then relationship (2.29) can be rewritten

\[ E_k = -\frac{2\mu_0 H_0^2}{\lambda_n ne} e^{-\frac{2\mu_0 H_0^2}{\lambda_n ne}}. \] (2.27)

Let us produce the estimate of the magnitude of this field for niobium, assuming \( n = 5.4 \cdot 10^{28} \text{ 1/m}^3 \). With the zero of temperatures the critical magnetic field of niobium is equal \( 1.5 \cdot 10^5 \text{ A/m} \). Substituting these values in relationship (2.27), we obtain the value of electrokinetic field near the surface \( E_k \approx 3 \cdot 10^7 \text{ V/m} \), when currents in superconductive niobium are close to the critical.

3. Conclusion

In the article is described the new physical of phenomenon, which indicates that in the superconductors the stationary electric field can exist. This field exists in the surface layer of superconductor and it is directed normal to its surface. Since the field indicated is the consequence of the kinetic motion of charges in the superconductor, it can be named electrokinetic electric field.

References