Langmuir frequency and its value for physics of the plasma

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Langmuir frequency by the very important electrodynamic parameter represents current resonance of displacement and conduction current with imposition on the plasma of variable electrical pour on, and it is introduced in a number of fundamental works on physics of the plasma [1-3].

Langmuir frequency is determined by the relationship

$$\omega_0 = \sqrt{\frac{1}{L_k \varepsilon_0}},$$

where ε_0 - the dielectric constant of the vacuum L_k - the kinetic inductance of the charges [4].

With the propagation in the plasma of electromagnetic waves the electrons play the dominant role, since their mass is considerably less than the mass of ions. The equation of motion of electron in the electric field takes the form:

$$m\frac{d\vec{v}}{dt} = e\vec{E}, \quad (1)$$

where m - the electron mass, e - the electron charge, $\vec{E}$ - the tension of electric field, $\vec{v}$ - speed of the motion of charge.

In the work [5] it is shown that this equation can be used also for describing the electron motion in the hot plasma. Therefore it can be disseminated also to this case.

Using an expression for the current density

$$\vec{j} = ne\vec{v}, \quad (2)$$

from (1) we obtain the current density of the conductivity

$$\vec{j}_L = \frac{ne^2}{m}\int \vec{E}\, dt. \quad (3)$$

In relationship (6.2) and (6.3) the value of represents electron density. After introducing the designation

$$L_k = \frac{m}{ne^2}, \tag{4}$$

we find

$$\vec{j}_L = \frac{1}{L_k} \int \vec{E}\, dt\,. \tag{5}$$

In this case the value L_k presents the specific kinetic inductance of charge carriers. Its existence connected with the fact that charge, having a mass, possesses inertia properties. Pour on relationship (6.5) it will be written down for the case of harmonics:

$$\vec{j}_L = -\frac{1}{\omega L_k} \vec{E}_0 \cos \omega t \tag{6}$$

From relationship (6.5) and (6.6) is evident that $\vec{j}_L$ presents inductive current, since its phase is late with respect to the tension of electric field to the angle $\frac{\pi}{2}$.

If charges are located in the vacuum, then during the presence of summed current it is necessary to consider bias current

$$\vec{j}_\varepsilon = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \varepsilon_0 \vec{E}_0 \cos \omega t\,.$$

Is evident that this current bears capacitive nature, since. its phase anticipates the phase of the tension of electrical to the angle $\frac{\pi}{2}$. Thus, summary current density will compose:

$$\vec{j}_\Sigma = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \frac{1}{L_k} \int \vec{E}\, dt\,,$$

or

$$\vec{j}_\Sigma = \left(\omega\varepsilon_0 - \frac{1}{\omega L_k} \right) \vec{E}_0 \cos\omega t = \omega\varepsilon_0 \left(1 - \frac{\omega_0^2}{\omega^2} \right) \vec{E}_0 \cos\omega t = \omega\varepsilon^*(\omega)\vec{E}_0 \cos\omega t \,. \tag{7}$$

The parameter $\varepsilon^*(\omega)$ to the literature, which is concerned plasma, is conventionally designated as the dielectric constant of plasma, although this and not thus. As can be seen from relationship (7) this is the composite parameter, into which besides frequency enter still several parameters.

Thus, with the presence in plasma of ac field in it simultaneously flow two currents of different nature: bias current and conduction current. These currents competing, since they have different dependence on the frequency of electric field. Equality to zero members in the brackets indicates the resonance of these currents.

How it is necessary to enter, if at our disposal are values $\varepsilon^*(\omega)$, and we should calculate the total specific energy, accumulated in the plasma with the presence in it of variable electrical pour on? Obviously, in this case one should consider not only energy of electrical pour on

$$W_E = \frac{1}{2}\varepsilon_0 E_0^2 ,$$

and kinetic energy of charge carriers

$$W_j = \frac{1}{2} L_k j_0^2 . \tag{8}$$

For this one should use relationship, given in the work

$$W_\Sigma = \frac{1}{2} \cdot \frac{d\left(\omega\varepsilon^*(\omega)\right)}{d\omega} E_0^2 , \tag{9}$$

from where we obtain

$$W_\Sigma = \frac{1}{2}\varepsilon_0 E_0^2 + \frac{1}{2} \; \frac{1}{\omega^2 L_k} \; E_0^2 = \frac{1}{2}\varepsilon_0 E_0^2 + \frac{1}{2} L_k j_0^2 \; .$$

The given relationships show that the specific energy consists of potential energy of electrical pour on and to kinetic energy of charge carriers.

with the examination of any media by our final task appears the presence of wave equation. In this case this problem is already practically solved.

Maxwell's equations for this case take the form:

$$rot\ \vec{E} = -\mu_0 \frac{\partial\ \vec{H}}{\partial\ t},$$
$$rot\ \vec{H} = \varepsilon_0 \frac{\partial\ \vec{E}}{\partial\ t} + \frac{1}{L_k}\int \vec{E}\ dt, \qquad (10)$$

where ε_0 and μ_0 - dielectric and magnetic constant of vacuum.

we obtain from (10):

$$rot\ rot\ \vec{H} + \mu_0\varepsilon_0 \frac{\partial^2 \vec{H}}{\partial\ t^2} + \frac{\mu_0}{L_k}\vec{H} = 0. \qquad (11)$$

For the case pour on, time-independent, equation (11) passes into the equation of London

$$rot\ rot\ \vec{H} + \frac{\mu_0}{L_k}\vec{H} = 0\ ,$$

where of $\lambda_L{}^2 = \dfrac{L_k}{\mu_0}$ - London depth of penetration.

It is interesting to examine the case, when electromagnetic wave is propagated between two conducting planes, between which is located the plasma [6].

Solution of system of equations (10) gives the possibility to determine the wave number:

$$k_z^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_\rho^2}{\omega^2}\right)\ , \qquad (12)$$

both the group and phase speeds

$$v_g^2 = c^2\left(1 - \frac{\omega_\rho^2}{\omega^2}\right), \qquad (13)$$

$$v_F^2 = \frac{c^2}{\left(1 - \frac{\omega_\rho^2}{\omega^2}\right)}, \tag{14}$$

where $c = \left(\frac{1}{\mu_0 \varepsilon_0}\right)^{1/2}$ - speed of light in the vacuum.

For the present instance the phase speed of electromagnetic wave is equal to infinity, which corresponds to transverse resonance at the plasma frequency. Consequently, at each moment of time pour on distribution and currents in this line uniform and it does not depend on the coordinate z, but current in the planes of line in the direction of is absent. This means that any planes or devices, which limit plasma from two sides, can be used instead of the conducting planes.

From relationships (12-14) is evident that at the point $\omega = \omega_p$ occurs the transverse resonance with the infinite quality. With the presence of losses in the resonator will occur the damping, and in the long line in this case of $k_z \neq 0$, and in the line will be extended the damped transverse wave, the direction of propagation of which will be normal to the direction of the motion of charges.

Until now, a physically unrealizable case has been considered, when there are no losses in the plasma, which corresponds to an infinite Q-factor of the plasma resonator.If losses are located, moreover completely it does not have value, by what physical processes such losses are caused, then the quality of plasma resonator will be finite quantity. For this case of Maxwell's equation they will take the form:

$$\begin{aligned} rot\ \vec{E} &= -\mu_0 \frac{\partial \vec{H}}{\partial t}, \\ rot\ \vec{H} &= \sigma_{p.ef}\ \vec{E} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \frac{1}{L_k} \int \vec{E}\ dt. \end{aligned} \tag{15}$$

the presence of losses is considered by the term $\sigma_{p.ef} \vec{E}$, and, using near the conductivity of the index of ef, it is thus emphasized that us does not interest very mechanism of losses, but only very fact of their existence interests. The value of σ_{ef} determines the quality of plasma resonator. For measuring σ_{ef} should be selected the section of line by the length z_0, whose value is considerably lower than the wavelength in the plasma. This section will be equivalent to outline with the lumped parameters:

$$C = \varepsilon_0 \frac{bz_0}{a}, \tag{16}$$

$$L = L_k \frac{a}{bz_0}, \tag{17}$$

$$G = \sigma_{\rho.ef} \frac{bz_0}{a}, \tag{18}$$

where G - conductivity, connected in parallel C and L.

Conductivity and quality in this outline enter into the relationship:

$$G = \frac{1}{Q_\rho}\sqrt{\frac{C}{L}},$$

from where, taking into account (16 – 18), we obtain:

$$\sigma_{\rho.ef} = \frac{1}{Q_\rho}\sqrt{\frac{\varepsilon_0}{L_k}}. \tag{19}$$

Thus, measuring its own quality plasma of the resonator examined, it is possible to determine $\sigma_{p.ef}$. Using (15) and (19) we will obtain:

$$\begin{aligned} rot\ \vec{E} &= -\mu_0 \frac{\partial \vec{H}}{\partial t}, \\ rot\ \vec{H} &= \frac{1}{Q_\rho}\sqrt{\frac{\varepsilon_0}{L_k}}\ \vec{E} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \frac{1}{L_k}\int \vec{E}\, dt. \end{aligned} \tag{20}$$

Let us examine the solution of system of equations (20) at the point $\omega = \omega_p$, in this case, since

$$\varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \frac{1}{L_k}\int \vec{E}\, dt = 0,$$

we obtain

$$rot\ \vec{E} = -\mu_0 \frac{\partial\ \vec{H}}{\partial\ t},$$

$$rot\ \vec{H} = \frac{1}{Q_P}\sqrt{\frac{\varepsilon_0}{L_k}}\ \vec{E}.$$

These relationships determine wave processes at the point of resonance, if in the plasma are losses.

If losses in the plasma, which fills line are small, and strange current source is connected to the line, then it is possible to assume:

$$rot\ \vec{E} \cong 0,$$

$$\frac{1}{Q_p}\sqrt{\frac{\varepsilon_0}{L_k}}\ \vec{E} + \varepsilon_0 \frac{\partial\ \vec{E}}{\partial\ t} + \frac{1}{L_k}\int \vec{E}\ dt = \vec{j}_{CT}, \tag{21}$$

where $\vec{j}_{CT}$ - density of strange currents.

Integrating (21) in time and dividing both sides by, we obtain

$$\omega_p^2\ \vec{E} + \frac{\omega_p}{Q_p}\cdot\frac{\partial\ \vec{E}}{\partial\ t} + \frac{\partial^2 \vec{E}}{\partial\ t^2} = \frac{1}{\varepsilon_0}\cdot\frac{\partial\ \vec{j}_{CT}}{\partial\ t}. \tag{22}$$

Если (22) проинтегрировать по поверхности нормальной к вектору $\vec{E}$ и ввести электрический поток как $\Phi_E = \int \vec{E} d\vec{S},$ получим:

If (22) is integrated over the surface normal to the vector $\vec{E}$ and the electric flux is introduced as $\Phi_E = \int \vec{E} d\vec{S},$ follows:

$$\omega_p^2 \Phi_E + \frac{\omega_p}{Q_p}\cdot\frac{\partial\ \Phi_E}{\partial\ t} + \frac{\partial^2 \Phi_E}{\partial\ t^2} = \frac{1}{\varepsilon_0}\cdot\frac{\partial\ I_{CT}}{\partial\ t}, \tag{23}$$

where I_{CT} is the external current.

Equation (23) is the equation of a harmonic oscillator with a right-hand side, characteristic of two-level lasers.

References

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