The principle of operation and the mathematical model of Van de Graaff generator

F. F. Mende, A. S. Dubrovin

Abstract

The physical principle of the work of Van de Graaff generator, until now, is not finally explained, but there is only a technical oscillator circuit. There are no calculated relationships, which give the possibility to calculate this generator. In the article physical oscillator circuit is represented and it is shown that the principle of its operation is based on the use of a law of parametric self-induction. Are obtained also the calculated relationships, which make it possible to calculate the parameters of generator.

The keywords: Van de Graaff generator, volt, capacity, parametric self-induction, pelletron.

1. Introduction

Van de Graaff generator (Fig. 1) this the generator of high voltage , the operating principle of which is based on the electrization of the moving dielectric tape. The first generator was developed by American physicist by Robert by Van de Graaff in 1929 the year and made it possible to obtain a potential difference to 80 the kilovolts . In 1931 1933 they built the more powerful generators, which made it possible to reach voltage in 1 million and 7 millions of volts respectively [1].

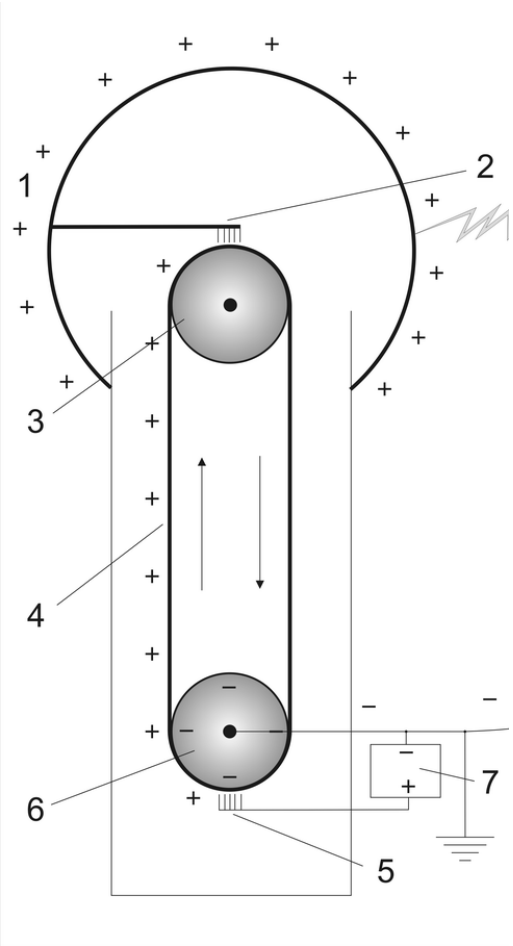

Fig. 1. Van de Graaff generator

Van de Graaff generator consists of the dielectric (silk or rubber) tape 4, of that revolving on the rollers 3 and 6, moreover upper roller dielectric, and lower metallic and is connected with the earth . One of the ends of the tape is concluded in the metallic sphere 1. Two electrodes 2 and 5 in the form brushes are located at small distance from the tape on top and from below, moreover electrode 2 is connected with the internal surface of the sphere 1. Through the brush 5 air is ionized from the source of high-voltage voltage 7, resultant positive ions under the action of Coulomb force they move to the grounded roller 6 they settle on the tape. The moving tape transfers charge inside the sphere 1, where it is removed by brush 2, under the action of Coulomb force charges they are pushed out to the surface of sphere and the field inside the sphere it is created only by booster charge on the

tape. Thus, on the external surface of sphere is accumulated electric charge. The possibility of obtaining the high voltage is limited by the corona discharge , appearing with the ionization of air around the sphere.

Contemporary Van de Graaff generators instead of the tapes use the chains, which consist of the alternating metallic and plastic it is sectional, which are called the pelletrons .

Unfortunately, the given oscillator circuit is only operational technical diagram, and the physical principle of its action, until now, causes number of questions. For example, it is incomprehensible, what reasons cause an increase in the potential of the charges, located on the tape, with its motion along the determinate direction. Generator will unclearly also function, if the moving tape was arranged the horizontally earth's surface. Incomprehensibly also, how can be changed the polarity of generator. But since neither the physical nor mathematical model of generator to the end is developed, its improvement can be carried out only by the trial-and-error method. With this is connected the circumstance that his construction practically did not change from the times of the invention of Van de Graaff generator.

2. Mathematical model of Van de Graaff generator

If there is a capacitor, whose capacity C , and this capacitor it is charged to a potential difference U , that the energy, accumulated in it, is determined by the relationship

$$W_C = \frac{1}{2} CU^2 . \tag{2.1}$$

But charge Q , accumulated in the capacity, is equal

$$Q_{C,U} = CU . \tag{2.2}$$

From relationship (2.1) it is evident that if the charge, accumulated in the capacity, remains constant, then voltage on it can be changed by changing the capacity. In this case is fulfilled the relationship

$$Q_{C,U} = CU = C_0U_0 = const,$$

where C, U - instantaneous values, and C_0, U_0 - initial values of these parameters.

The represented relationship presents the law of the parametric self-induction [3-6].

The voltage on the capacity and the energy, accumulated in it, will be in this case determined by the relationships:

$$U = \frac{C_0U_0}{C} = KU_0, \tag{2.3}$$

$$W_C = \frac{1}{2}\ \frac{\left(C_0U_0\right)^2}{C}. \tag{2.4}$$

Coefficient

$$K = \frac{C_0}{C} \tag{2.5}$$

let us name the transformation ratio of constant voltage. It is easy this coefficient by the passing track of changing the relation of capacities.

The incremental voltage, which can ensure this transformer, is determined from the relationship

$$\Delta U_C = \left(\frac{C_0}{C} - 1\right)U_0. \tag{2.6}$$

As follows from the relationships (2.3) and (2.4) with the decrease of capacitance of capacitor on it increases not only voltage, but also the energy, stored up in the Ger. This energy is selected in the mechanical energy source, which ensures a change in the capacity. Therefore the transformer in question can be considered, and as the converter of mechanical energy into the electrical.

An increase in the energy, accumulated in the capacitor, with a change in its capacity is determined from the relationship

$$\Delta W_C = \frac{1}{2}\left(C_0 U_0\right)^2 \left(\frac{1}{C} - \frac{1}{C_0}\right). \qquad (2.7)$$

Relationships (2.3-2.7) determine physics of the work of Van de Graaff generator. The moving metallic pelletrons of tape or its sections have relative to the earth a capacity, which during the motion of these sections relative to the earth changes according to the specific law. In the base of generator these sections should be loaded to the assigned potential of the specific sign. If the capacity of these sections will change relative to the earth, then will change the potential of the charges, located on them. In the upper part of the generator these sections betray the accumulated potential to sphere, charging it to the high potential.

For calculating the generator it is necessary to know the initial potential of metallic pelletron and the law of variation in their capacity with respect to the earth during the motion of tape. Should be also known the extent of their movement from the lower part of the generator, where they are charged, to its upper part, where they return their charge to sphere. Therefore in this case the main mathematical problem of calculating the generator is the presence of the dependence of capacity pelletron depending on distance to the earth. With the vertical position of generator this there will be one dependence, with the horizontal position - another. If tape moves in parallel to the earth, then this dependence will be absent, and generator work will not be. The metallic grounded elements of the construction of building also must be absent near the vertical part of the generator.

The precise calculation of capacity pelletron relative to the earth to perform difficultly, but a good approximation is assumption about the fact that the pelletrons present the conducting spheres, whose diameter is equal to its size. In this case it is necessary to calculate the capacity of the sphere of the intended size relative to the flat conducting surface, which is the earth. This dependence is known and is determined by the formula (2,7)

$$C = 4\pi\varepsilon a \sum_{n=1}^{\infty} \frac{\sinh\left[\ln\left(D+\sqrt{D^2-1}\right)\right]}{\sinh\left[n\ln\left(D+\sqrt{D^2-1}\right)\right]} = \quad , \qquad (2.8)$$

$$= 4\pi\varepsilon a\left(1+\frac{1}{2D}+\frac{1}{4D^2}+\frac{1}{8D^3}+\frac{1}{32D^5}+\ .....\right)$$

where $D=\frac{d}{2a}$, a - a radius of sphere, d - distance from the lower part of the generator to its upper part.

The first term in the decomposition (2.8) represents the capacity of the secluded sphere and does not depend from the distance to the earth. Us they will interest only that capacity, which depends on distance.

In the case, when d it is considerably more than a in the relationship (2.8) it suffices to take only second term of expansion. In this case the dependence of capacity pelletron on the distance to be determined by the relationship

$$C = 4\pi\varepsilon\frac{a^2}{d}. \qquad (2.9)$$

In the lower position of pelletron its capacity relative to the earth comprises

$$C_0 = 4\pi\varepsilon\frac{a^2}{d_0}, \qquad (2.10)$$

where d_0 - the distance of pelletron to the earth in the lower position.

Consequently, the transformation ratio of potential can be found from the relationship (2.5)

$$K = \frac{d}{d_0}$$

Thus, are acquired all necessary data for designing the generator. The practical oscillator circuit, in which are taken into account the principles examined, is represented in Fig. 2

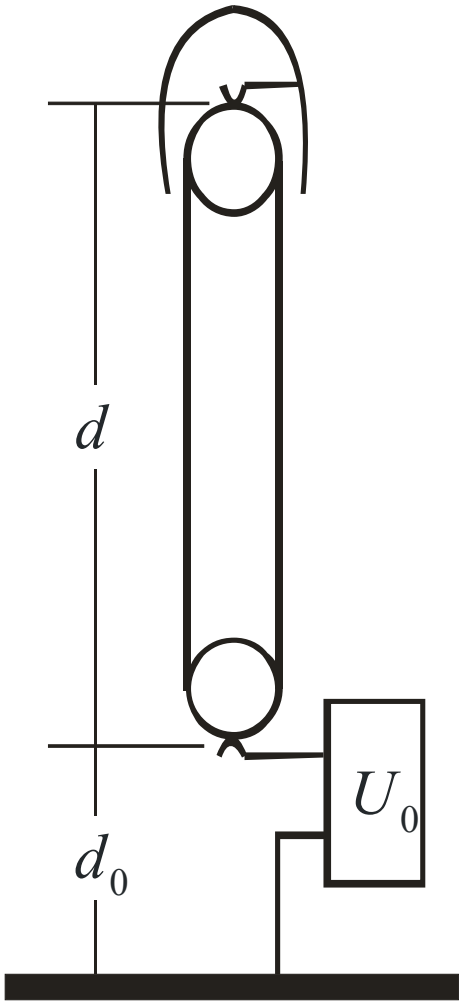

Fig. 2 Oscillator circuit

In contrast to the construction, given in Fig. 1, both upper and lower roller they are made from dielectric, and lower and upper brushes slide along the belt with peletron. Each peletron, moving around the lower roller, by means of the brush is charged from the voltage source U_0. On the polarity of this source depends the polarity of the voltage, manufactured by generator.

In order to increase transformation ratio, one should decrease d_0. With this purpose lower roller can be made composite. Its internal part should be carried out from the metal and grounded, and outside dressed collar from rubber or cylinder from the dielectric. In this case the thickness of collar or cylinder will be the size

d_0. It is possible to enter and differently. Roller to make completely from the metal and to ground, and on the rubber tape of peletron to apply the way of metallization. Then the thickness d_0 of belt will serve as size.

Earlier us lacked the possibility to calculate voltage and power of Van de Graaff generator, now this possibility is located. For this it follows to use relationships (2.3) and (2.7).

Let us give concrete example. At our disposal there is a metallized rubber tape by the thickness d_0 of the equal 0.01 m and by the width 0.1 m, which corresponds to a radius a of equivalent sphere 0.05 m. On this tape there are metallized square sections (peletrony), which alternate with the same not metallized sections. The speed of belt 50 m/s, the distance between the lower and upper brushes d are 5 m, the voltage of the voltage source U_0 is equal 10^4 V.

The voltage, generated by generator, will comprise with the parameters indicated

$$U = \frac{d}{d_0} U_0 = 5000 \ kV .$$

With the speed of belt 50 m/s in second the charge to upper brush will return 250 peletrons. Each peletron will return the energy equal, determined by the relationship (2.4). After using relationships (2.9) (2.10) we obtain the generatable power

$$P = 500\pi\varepsilon \frac{a^2 d}{d_0^2} U_0^2 .$$

Calculation according to this formula taking into account the given parameters gives power 174 W.

Using relationships (2.1) and (2.4) it is possible to calculate the efficiency of the generator, which is equal to the ratio of the manufactured energy to the energy, expended by the voltage source. In this case efficiency it will be equal

$$eff = \frac{d^2}{d_0^2}.$$

With the parameters efficiency indicated it composes the value 2.5×10^{4} . This high efficiency means that practically entire mechanical energy (if we do not consider energy consumption for the drive of the motion of tape) it is expended on the production of electrical energy. By this high efficiency possesses none of the existing generators of electric power. The question about the creation of competitorily capable generator in comparison with the existing generators naturally arises. This can be solved only by the way of an increase in the capacitance of peletron relative to the earth. There is this way. Yes, this way is located. For this it is necessary to use the experience of the creation of the cermet capacitors, in which as the dielectric is used barium titanate, which has very high dielectric constant. The schematic of this generator is represented in Fig. 3. The represented diagram is the analog of Man de Graffss generator.

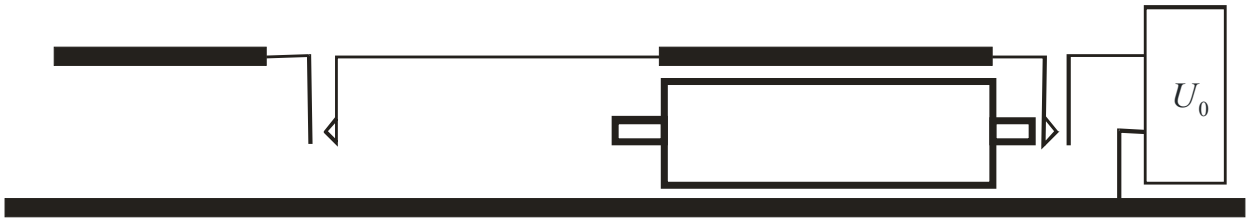

Fig. 3. Schematic of a generator

In the diagram fatty sections designated the facings of parallel-plate capacitors. Heavy solid line designated the lower facing, which is general for both capacitors. The earth can be this facing. Between the facings of left capacitor is located the plate from barium titanate, at ends of which are protrusions, with the aid of which can be locked and be opened contact pairs. When plate is located in the extreme by right position, it locks the contact pair, which connects the voltage source with the upper plate of right capacitor, charging it to the potential U_0 . In this position of

plate the capacitance of right capacitor is maximum. When plate begins to be moved to the left, right contact pair is opened, disconnecting capacitor from the voltage source. During further motion of plate the capacitance of right capacitor begins to decrease according to the linear law and potential on it grows. In the end left situation, when plate exceeds the limits of right of capacitor, and potential on it reaches maximum value, occurs closing left contact pair and part from the left capacitor passes charge into the right capacitor, and their potentials are equalized. Further cycle is repeated with the return of plate to the end right position. Thus, the transformation of potential in this case occurs according to the already examined above diagram.

Conclusion

The physical principle of the work of Van de Graaff generator, until now, is not finally described, but there is only a technical oscillator circuit. There are no calculated relationships, which give the possibility to calculate this generator. In the article physical oscillator circuit is represented and it is shown that the principle of its operation is based on the use of a law of parametric self-induction. Are obtained also the calculated relationships, which make it possible to calculate the parameters of generator. The calculation, carried out employing the proposed procedure, shows that the generator in question possesses the very high efficiency, which is not accessible in the existing generators.

References

1. Darryl J. Leiter. Van de Graaff, Robert Jemison // A to Z of Physicists, 2003, p. 312.

2. Maxwell J. C. A Treatise on Electricity and Magnetism. — Dover, 1873. — P. 266 ff.

3. F. F. Mende, A. S. Dubrovin. Special properties of reactive elements and fluxes of charged particles. Engineering Physics, No. 11, 2016, p. 13-21

4. F. F. Mende, New Properties of Reactive Elements and the Problem of Propagation of Electrical Signals in Long Lines, American Journal of Electrical and Electronic Engineering, Vol. 2, No. 5, (2014), p.141-145.

5. F. F. Mende. Induction and Parametric Properties of Radio-Technical Elements and Lines and Property of Charges and Their Flows, AASCIT Journal of Physics Vol.1 , No. 3, Publication Date: May 21, 2015, p. 124-134

6. F. F. Mende, A. S. Dubrovin. Alternative ideology of electrodynamics. M .: Perot, 2016. - 198 p.

7. A. D. Rawlins. Note on the Capacitance of Two Closely Separated Spheres. IMA Journal of Applied Mathematics v. 34 (1): 1985 p. 119–120.