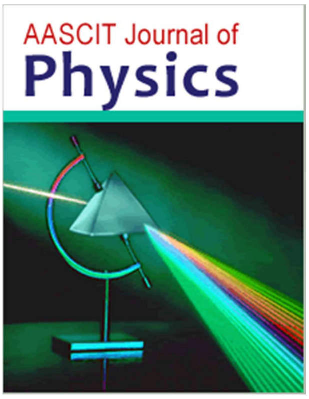
AASCIT Journal of Physics
2015; 1(2): 69-75
Published online April 30, 2015 (http://www.aascit.org/journal/physics)

American Association for Science and Technology



Keywords
Coulomb's Law,
Scalar Potential,
Vector Potential,
Laws of Induction

Received: March 30, 2015
Revised: April 14, 2015
Accepted: April 15, 2015

Classical Relativistic Corrections to Coulomb Law

F. F. Mende

B. Verkin Institute for Low Temperature Physics and Engineering NAS Ukraine, Kharkov, Ukraine

Email address
mende_fedor@mail.ru

Citation
F. F. Mende. Classical Relativistic Corrections to Coulomb Law. *AASCIT Journal of Physics.* Vol. 1, No. 2, 2015, pp. 69-75.

Abstract
Coulomb's law determines the static force, which acts between two fixed charges. However, this law does not give the answer, how this force will change, if charges will accomplish relative motion. Answer to this question can be obtained from the laws of induction. In the article it is shown that also the scalar potential and the electric field of charge at the observation point depend on relative motion of charge relative to this point.

1. Introduction

Coulomb law determines the static force, which acts between two fixed charges [1]

$$\vec{F}_1 = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}^2}\vec{e}_{12}, \tag{1.1}$$

where $\vec{F}_1$ is force, which acts on the charge q_1, $\vec{e}_{12}$ is unit vector, directed from the charge q_2 toward q_1, r_{12} is distance between the charges.

The scalar potential of the charge follows from this law

$$\varphi = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \tag{1.2}$$

But the field strength, created by charge, it is determined from the relationship

$$E = -grad\ \varphi \tag{1.3}$$

However, Coulomb law does not give answer to a question about how to change scalar potential and the tension of the electric field of charge during its motion answer to this question can be obtained from the laws of induction and Ampere law.

2. Laws of the Magneto Electric Induction

The important task of electrodynamics is the presence of laws governing the appearance of electrical pour on, and, therefore, also the forces of those acting on the charge, at the particular point spaces, since. Only electric fields, generated other one or method or another, exert power influences on the charge. Such fields can be obtained, changing the arrangement of other charges around this point of space or accelerating these charges. If around the point in question is some static configuration of charges, then the tension of electric field will be at the particular point determined by the relationship of , where the scalar potential at the assigned point, determined by the assigned configuration of charges. If we change the arrangement of charges, then this new configuration will correspond other values of scalar potential, and, therefore, also other values of the tension of electric field. But, making this, it is necessary to move charges in the space, and this displacement in the required order is cmbined with their acceleration and subsequent retarding. Acceleration or retarding of charges also can lead to the appearance in the surrounding space of induction electrical pour on. Can arise another stationary situation, when after their acceleration charges move in the environment of the considered point with the constant velocity along circular or other locked trajectories. In this case due to the presence of the three-dimensional velocity gradients in the flows of the moving charges configurative electric fields can appear.

In the electrodynamics the fundamental law of induction is Faraday law. In the contemporary electrodynamics it is written as follows:

$$\oint \vec{E}\, d\vec{l} = -\frac{\partial \Phi_B}{\partial t} = -\mu \int \frac{\partial \vec{H}}{\partial t}\, d\vec{s} = -\int \frac{\partial \vec{B}}{\partial t}\, d\vec{s}\ , \quad (2.1)$$

where $\vec{B} = \mu \vec{H}$ is magnetic induction vector, $\Phi_B = \mu \int \vec{H}\, d\vec{s}$ is flow of magnetic induction, and $\mu = \tilde{\mu}\mu_0$ is magnetic permeability of medium. It follows from this law that the circulation integral of the vector of electric field is equal to a change in the flow of magnetic induction through the area, which this outline covers. It is immediately necessary to emphasize the circumstance that the law in question presents the processes of mutual induction, since. For obtaining the circulation integral of the vector $\vec{E}$ we take the strange magnetic field, formed by strange source. From relationship (2.1) obtain the Maxwell first equation

$$rot\ \vec{E} = -\frac{\partial \vec{B}}{\partial t}\ . \quad (2.2)$$

Let us immediately point out to the terminological error. Faraday law should be called not the law of electromagnetic, as is customary in the existing literature, but by the law of magnetoelectric induction, since. a change in the magnetic pour on it leads to the appearance of electrical pour on, but not vice versa.

Let us introduce the vector potential of the magnetic field $\vec{A}_H$, which satisfies the equality

$$\mu \oint \vec{A}_H\, d\vec{l} = \Phi_B\ ,$$

where the outline of the integration coincides with the outline of integration in relationship (2.1), and the vector of is determined in all sections of this outline, then

$$\vec{E} = -\mu \frac{\partial \vec{A}_H}{\partial t} \quad (2.3)$$

introduced thus vector $\vec{A}_H$ determines the local connection between it and by electric field, and also between the gradients this vector and the magnetic field. If it will be possible to determine the vector $\vec{A}_H$, its time derivative at any point of space, and also its gradients, then will succeed in determining the vector $\vec{E}$, and the vector $\vec{H}$. It is not difficult to show that introduced thus vector $\vec{A}_H$, is connected with the magnetic field with the following relationship:

$$rot\ \vec{A}_H = \vec{H}\ . \quad (2.4)$$

If the discussion deals with the motion in the field of the three-dimensional changing vector potential, then for finding the induced electrical pour on should be used total derivative [2-8].

$$\vec{E}' = -\mu \frac{d\vec{A}_H}{dt} \quad (2.5)$$

prime near the vector $\vec{E}$ means that we determine this field in the moving coordinate system. This means that the vector potential can have not only local, but also convection derivative, i.e., it can change both due to the change in the time and due to the motion in the three-dimensional changing field of this potential. In this case relationship (2.5) can be rewritten as follows:

$$\vec{E}' = -\mu \frac{\partial \vec{A}_H}{\partial t} - \mu\left(\vec{v}\nabla\right)\vec{A}_H\ ,$$

where $\vec{v}$ is speed of the prime system. Consequently, the force, which acts on the charge in the moving system, in the absence the dependence of vector potential on the time, will be written down

$$\vec{F}'_{v,1} = -\mu e\left(\vec{v}\nabla\right)\vec{A}_H\ .$$

This force depends only on the gradients of vector potential and charge rate.

The charge, which moves in the field of the vector potential $\vec{A}_H$ with the speed $\vec{v}$, possesses potential energy [1]

$$W = -e\mu\left(\vec{v}\vec{A}_H\right).$$

Therefore must exist one additional force, which acts on the charge in the moving coordinate system, namely:

$$\vec{F}'_{v,2} = -grad\ W = e\mu\ grad\left(\vec{v}\vec{A}_H\right).$$

Thus, the value $e\mu\left(v\vec{A}_H\right)$ plays the same role, as the scalar potential φ, whose gradient also gives force. Consequently, the composite force, which acts on the charge, which moves in the field of vector potential, can have three components and will be written down as

$$\vec{F}' = -e\mu \frac{\partial \vec{A}_H}{\partial t} - e\mu\left(\vec{v}\nabla\right)\vec{A}_H + e\mu\ grad\left(\vec{v}\vec{A}_H\right). \quad (2.6)$$

The first of the components of this force acts on the fixed charge, when vector potential changes in the time and has local time derivative. Second component is connected with the motion of charge in the three-dimensional changing field of this potential. Entirely different nature in force, which is determined by last term of relationship (2.6). It is connected with the fact that the charge, which moves in the field of vector potential, it possesses potential energy, whose gradient gives force. From relationship (2.6) follows

$$\vec{E}' = -\mu \frac{\partial \vec{A}_H}{\partial t} - \mu(\vec{v}\nabla)\vec{A}_H + \mu \; grad\left(\vec{v}\vec{A}_H\right). \qquad (2.7)$$

This is a complete law of mutual induction. It defines all electric fields, which can appear at the assigned point of space, this point can be both the fixed and that moving. This united law includes and Faraday law and that part of the Lorentz force, which is connected with the motion of charge in the magnetic field, and without any exceptions gives answer to all questions, which are concerned mutual magnetoelectric induction. It is significant, that, if we take rotor from both parts of equality (2.7), attempting to obtain the Maxwell first equation, then it will be immediately lost the essential part of the information, since rotor from the gradient is identically equal to zero.

If we isolate those forces, which are connected with the motion of charge in the three-dimensional changing field of vector potential, and to consider that

$$\mu \; grad\left(\vec{v}\vec{A}_H\right) - \mu(\vec{v}\nabla)\vec{A}_H = \mu\left[\vec{v}\times rot\;\vec{A}_H\right],$$

that from (2.6) we will obtain

$$\vec{F}'_v = e\mu\left[\vec{v}\times rot\;\vec{A}_H\right], \qquad (2.8)$$

and, taking into account (2.4), let us write down

$$\vec{F}'_v = e\mu\left[\vec{v}\times\vec{H}\right], \qquad (2.9)$$

or

$$\vec{E}'_v = \mu\left[\vec{v}\times\vec{H}\right], \qquad (2.10)$$

and it is final

$$\vec{F}' = e\vec{E} + e\vec{E}'_v = -e\frac{\partial \vec{A}_H}{\partial t} + e\mu\left[\vec{v}\times\vec{H}\right]. \qquad (2.11)$$

Can seem that relationship (2.11) presents Lorentz force; however, this not thus. In this relationship and field $\vec{E}$, and field $\vec{E}'_v$ they are induced. The first field is connected with the presence of the local derivative of vector potential on the time, the second is obliged to the presence of the convective derivative of this potential. In order to obtain the total force, which acts on the charge, necessary to the right side of relationship (2.11) to add the term $-e\; grad\;\varphi$

$$\vec{F}'_\Sigma = -e\; grad\;\varphi + e\vec{E} + e\mu\left[\vec{v}\times\vec{H}\right],$$

where φ is scalar potential at the observation point. In this case relationship (2.7) can be rewritten as follows:

$$\vec{E}' = -\mu \frac{\partial \vec{A}_H}{\partial t} - \mu(\vec{v}\nabla)\vec{A}_H + \mu \; grad\left(\vec{v}\vec{A}_H\right) - grad\;\varphi, \qquad (2.12)$$

or, after writing down the first two members of the right side of relationship (2.12) as the derivative of vector potential on the time, and also, after introducing under the sign of gradient two last terms, we will obtain

$$\vec{E}' = -\mu \frac{d\vec{A}_H}{dt} + grad\left(\mu\left(\vec{v}\vec{A}\right) - \varphi\right). \qquad (2.13)$$

If both parts of relationship (2.12) are multiplied by the magnitude of the charge, then will come out the total force, which acts on the charge. From Lorentz force it will differ in terms of the force $-e\mu\frac{\partial \vec{A}_H}{\partial t}$. From relationship (2.13) it is evident that the value $\mu\left(\vec{v}\vec{A}\right) - \varphi$ plays the role of the generalized scalar potential. Let us examine, what fields determine the first member of the right side of relationship (2.13).

The Ampere law, expressed in the vector form, determines magnetic field at the point x, y, z [9]

$$\vec{H} = \frac{1}{4\pi}\int \frac{I d\vec{l}\times\vec{r}}{r^3},$$

where I is current in the element $d\vec{l}$, $\vec{r}$ is vector, directed from $d\vec{l}$ to the point x, y, z .

It is possible to show that

$$\frac{[d\vec{l}\vec{r}]}{r^3} = grad\left(\frac{1}{r}\right)\times d\vec{l},$$

and, besides the fact that

$$grad\left(\frac{1}{r}\right)\times d\vec{l} = rot\left(\frac{d\vec{l}}{r}\right) - \frac{1}{r} rot\; d\vec{l}.$$

but the rotor $d\vec{l}$ is equal to zero and therefore is final

$$\vec{H} = rot\int I\left(\frac{d\vec{l}}{4\pi r}\right) = rot\;\vec{A}_H,$$

where

$$\vec{A}_H = \int I\left(\frac{d\vec{l}}{4\pi r}\right). \qquad (2.14)$$

The remarkable property of this expression is that that the vector potential depends from the distance to the observation point as $\frac{1}{r}$. Specifically, this property makes it possible to obtain emission laws.

Since $I = gv$, where g linear charge, from relationship (2.14) we obtain:

$$\vec{A}_H = \int \frac{gv\, d\vec{l}}{4\pi r}.$$

for the single charge of e this relationship takes the form:

$$\vec{A}_H = \frac{e\vec{v}}{4\pi r}. \quad (2.15)$$

Therefore the first member of the right side of relationship (2.13) is different from zero when the moving charge it has an acceleration.

However the member of $\mu\left(\vec{v}\vec{A}\right)$ in the right side of relationship (2.13) even with the constant velocity of charge has the specific value.

Let us determine the generalized (summary) potential of $\mu\left(\vec{v}\vec{A}\right)-\varphi$. Taking into account relationship (1.3) from (2.15) we obtain

$$\varphi_\Sigma = \mu\left(\vec{v}\vec{A}\right)-\varphi = \frac{\mu e v_\perp^2}{4\pi r} - \frac{e}{4\pi\varepsilon_0 r} = \frac{e v_\perp^2}{4\pi r \varepsilon_0 c} - \frac{e}{4\pi\varepsilon_0 r}. \quad (2.16)$$

In this relationship $v_\perp$ is speed normal to the vector, which connects charges, and is also taken into account that that $c^2 = \frac{1}{\mu\varepsilon_0}$.

In the case of relative motion of charges relationship (2.16) takes the form:

$$\varphi_\Sigma = \mu\left(\vec{v}\vec{A}\right)-\varphi = -\varphi\left(1+\frac{v_\perp^2}{c^2}\right). \quad (2.17)$$

Thus, the scalar potential of the charge, which accomplishes motion relative to observation point, depends on its speed.

On speed depends the created by it electric field

$$E = -grad\ \varphi_\Sigma = \frac{e}{4\pi\varepsilon_0 r}\left(1+\frac{v_\perp^2}{c^2}\right).$$

This law, obtained from the law of the induction Faraday and Ampere law, he indicates that the field of the moving charge is differed from the field of fixed. However, this law is subject to refinement taking into account the law of electromagnetic induction.

3. Laws of the Electromagnetic Induction

Faraday law shows, how a change in the magnetic pour on it leads to the appearance of electrical pour on. However, does arise the question about that, it does bring a change in the electrical pour on to the appearance of any others pour on and, in particular, magnetic? Maxwell gave answer to this question, after introducing bias current into its second equation. In the case of the absence of conduction currents the second equation of Maxwell appears as follows:

$$rot\ \vec{H} = \varepsilon\frac{\partial\vec{E}}{\partial t} = \frac{\partial\vec{D}}{\partial t},$$

where $\vec{D} = \varepsilon\vec{E}$ is electrical induction.

From this relationship it is not difficult to switch over to the expression

$$\oint \vec{H}\ d\vec{l} = \frac{\partial\Phi_E}{\partial t}, \quad (3.1)$$

where $\Phi_E = \int \vec{D}\ d\vec{s}$ the flow of electrical induction.

However for the complete description of the processes of the mutual electrical induction of relationship (3.1) is insufficient. As in the case Faraday law, should be considered the circumstance that the flow of electrical induction can change not only due to the local derivative of electric field on the time, but also because the outline, along which is produced the integration, it can move in the three-dimensional changing electric field. This means that in relationship (3.1), as in the case Faraday law, should be replaced the partial derivative by the complete. Designating by the primes of field and circuit elements in moving inertial system [IS], we will obtain:

$$\oint \vec{H}'d\vec{l}' = \frac{d\Phi_E}{dt},$$

and further

$$\oint \vec{H}'d\vec{l}' = \int\frac{\partial\vec{D}}{\partial t}\ d\vec{s}' + \oint\left[\vec{D}\times\vec{v}\right]\ d\vec{l}' + \int\vec{v}\ div\vec{D}\ d\vec{s}'. \quad (3.2)$$

For the electrically neutral medium $div\vec{E} = 0$; therefore the last member of right side in this expression will be absent. For this case relationship (3.2) will take the form:

$$\oint \vec{H}'d\vec{l}' = \int\frac{\partial\vec{D}}{\partial t}\ d\vec{s}' + \oint\left[\vec{D}\times\vec{v}\right]\ d\vec{l}'. \quad (3.3)$$

4. Dynamic Potentials and the Field of the Moving Charges

If the law of induction (2.1) was written down in the total derivatives, then we will obtain

$$\oint \vec{E}'dl' = -\int\frac{\partial\vec{B}}{\partial t}d\vec{s} + \oint\left[\vec{v}\times\vec{B}\right]dl'.$$

Together with law (3.3) it presents two symmetrical laws of induction [2-5]

$$\oint \vec{E}'dl' = -\int \frac{\partial \vec{B}}{\partial t}d\vec{s} + \oint \left[\vec{v}\times\vec{B}\right]dl' \quad , \qquad (4.1)$$
$$\oint \vec{H}'dl' = \int \frac{\partial \vec{D}}{\partial t}d\vec{s} - \oint \left[\vec{v}\times\vec{D}\right]dl'$$

or

$$rot\vec{E}' = -\frac{\partial \vec{B}}{\partial t} + rot\left[\vec{v}\times\vec{B}\right] \quad . \qquad (4.2)$$
$$rot\vec{H}' = \frac{\partial \vec{D}}{dt} - rot\left[\vec{v}\times\vec{D}\right]$$

For the constants fields on these relationships they take the form:

$$\vec{E}' = \left[\vec{v}\times\vec{B}\right]$$
$$\vec{H}' = -\left[\vec{v}\times\vec{D}\right] . \qquad (4.3)$$

In relationships (4.1-4.3), which assume the validity of the Galileoconversions, prime and not prime values present fields and elements in moving and fixed ISrespectively. It must be noted, that conversions (4.3) earlier could be obtained only from the conversions of Lorenz.

The relationships (4.1-4.3), which present the laws of induction, do not give information about how arose fields in initial fixed IS. They describe only laws governing the propagation and conversion pour on in the case of motion with respect to the already existing fields.

The relationship (4.3) attest to the fact that in the case of relative motion of frame of references, between the fields $\vec{E}$ and $\vec{H}$ there is a cross coupling, i.e., motion in the fields of $\vec{H}$ leads to the appearance pour on $\vec{E}$ and vice versa. From these relationships escape the additional consequences, which were for the first time examined in the work [2].

The electric field $E = \frac{g}{2\pi\varepsilon r}$ outside the charged long rod with a linear density g decreases as $\frac{1}{r}$, where r is distance from the central axis of the rod to the observation point.

If we in parallel to the axis of rod in the field E begin to move with the speed Δv another IS, then in it will appear the additional magnetic field $\Delta H = \varepsilon E\Delta v$. If we now with respect to already moving IS begin to move third frame of reference with the speed Δv, then already due to the motion in the field ΔH will appear additive to the electric field $\Delta E = \mu\varepsilon E\left(\Delta v\right)^2$. This process can be continued and further, as a result of which can be obtained the number, which gives the value of the electric field $E_v'\left(r\right)$ in moving IS with reaching of the speed $v = n\Delta v$, when $\Delta v \to 0$, and $n \to \infty$. In the final analysis in moving IS the value of dynamic electric field will prove to be more than in the initial and to be determined by the relationship:

$$E'\left(r, v_\perp\right) = \frac{gch\frac{v_\perp}{c}}{2\pi\varepsilon r} = Ech\frac{v_\perp}{c} .$$

If speech goes about the electric field of the single charge e , then its electric field will be determined by the relationship:

$$E'\left(r, v_\perp\right) = \frac{ech\frac{v_\perp}{c}}{4\pi\varepsilon r^2} ,$$

where $v_\perp$ is normal component of charge rate to the vector, which connects the moving charge and observation point.

Expression for the scalar potential, created by the moving charge, for this case will be written down as follows:

$$\varphi'(r, v_\perp) = \frac{ech\frac{v_\perp}{c}}{4\pi\varepsilon r} = \varphi(r)ch\frac{v_\perp}{c} \qquad (4.4)$$

where $\varphi(r)$ is scalar potential of fixed charge. The potential $\varphi'(r, v_\perp)$ can be named scalar- vector, since. it depends not only on the absolute value of charge, but also on speed and direction of its motion with respect to the observation point. Maximum value this potential has in the direction normal to the motion of charge itself. Moreover, if charge rate changes, which is connected with its acceleration, then can be calculated the electric fields, induced by the accelerated charge.

During the motion in the magnetic field, using the already examined method, we obtain:

$$H'(v_\perp) = Hch\frac{v_\perp}{c} .$$

where $v_\perp$ is speed normal to the direction of the magnetic field.

If we apply the obtained results to the electromagnetic wave and to designate components fields on parallel speeds IS as $E_\uparrow$ and $H_\uparrow$, and $E_\perp$ and $H_\perp$ as components normal to it, then with the conversion fields on components, parallel to speed will not change, but components, normal to the direction of speed are converted according to the rule

$$\vec{E}_\perp' = \vec{E}_\perp ch\,\frac{v}{c} + \frac{v}{c}\vec{v}\times\vec{B}_\perp sh\frac{v}{c},$$
$$\vec{B}_\perp' = \vec{B}_\perp ch\,\frac{v}{c} - \frac{1}{vc}\vec{v}\times\vec{E}_\perp sh\frac{v}{c}, \qquad (4.5)$$

where c is speed of light.

Conversions fields (4.5) they were for the first time obtained in the work [2].

However, the iteration technique, utilized for obtaining the given relationships, it is not possible to consider strict, since its convergence is not explained

Let us give a stricter conclusion in the matrix form [10].

Let us examine the totality IS of such, that IS K_1moves with the speed Δv relative to IS K, ISK_2 moves with the same speed Δv relative to K_1, etc. If the module of the speed Δv is small (in comparison with the speed of light c), then for the transverse components fields on in ISK_1, K_2,…. we have:

$$\vec{E}_{1\perp} = \vec{E}_\perp + \Delta\vec{v}\times\vec{B}_\perp \qquad \vec{B}_{1\perp} = \vec{B}_\perp - \Delta\vec{v}\times\vec{E}_\perp / c^2$$
$$\vec{E}_{2\perp} = \vec{E}_{1\perp} + \Delta\vec{v}\times\vec{B}_{1\perp} \qquad \vec{B}_{2\perp} = \vec{B}_{1\perp} - \Delta\vec{v}\times\vec{E}_{1\perp} / c^2 . \quad (4.6)$$

Upon transfer to each following IS of field are obtained increases in $\Delta\vec{E}$ and $\Delta\vec{B}$

$$\Delta\vec{E} = \Delta\vec{v}\times\vec{B}_\perp, \qquad \Delta\vec{B} = -\Delta\vec{v}\times\vec{E}_\perp / c^2 , \quad (4.7)$$

where of the field $\vec{E}_\perp$ and $\vec{B}_\perp$ relate to current IS. Directing Cartesian axis x along $\Delta\vec{v}$, let us rewrite (4.7) in the components of the vector

$$\Delta E_y = -B_z\Delta v, \quad \Delta E = B_y\Delta v, \quad \Delta B_y = E_z\Delta v / c^2 . \quad (4.8)$$

Relationship (4.8) can be represented in the matrix form

$$\Delta U = AU\Delta v \quad \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1/c^2 & 0 & 0 \\ -1/c^2 & 0 & 0 & 0 \end{pmatrix} \quad U = \begin{pmatrix} E_y \\ E_z \\ B_y \\ B_z \end{pmatrix}.$$

If one assumes that the speed of system is summarized for the classical law of addition of velocities, i.e. the speed of final IS $K' = K_N$ relative to the initial system K is $v = N\Delta v$, then we will obtain the matrix system of the differential equations of

$$\frac{dU(v)}{dv} = AU(v) , \quad (4.9)$$

with the matrix of the system v independent of the speed A. The solution of system is expressed as the matrix exponential curve $\exp(vA)$:

$$U' \equiv U(v) = \exp(vA)U, \qquad U = U(0) , \quad (4.10)$$

here U is matrix column fields on in the system K, and U' is matrix column fields on in the system K'. Substituting (4.10) into system (4.9), we are convinced, that U' is actually the solution of system (4.9):

$$\frac{dU(v)}{dv} = \frac{d\left[\exp(vA)\right]}{dv}U = A\exp(vA)U = AU(v) .$$

It remains to find this exponential curve by its expansion in the series:

$$\exp(va) = E + vA + \frac{1}{2!}v^2A^2 + \frac{1}{3!}v^3A^3 + \frac{1}{4!}v^4A^4 + ...$$

where E is unit matrix with the size 4×4. For this it is convenient to write down the matrix A in the unit type form

$$A = \begin{pmatrix} 0 & -\alpha \\ \alpha/c^2 & 0 \end{pmatrix}, \qquad \alpha = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad 0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

then

$$A^2 = \begin{pmatrix} -\alpha^2/c^2 & 0 \\ 0 & -\alpha/c^2 \end{pmatrix}, \qquad A^3 = \begin{pmatrix} 0 & \alpha^3/c^2 \\ -\alpha^3/c^4 & 0 \end{pmatrix},$$

$$A^4 = \begin{pmatrix} \alpha^4/c^4 & 0 \\ 0 & \alpha^4/c^4 \end{pmatrix}, \qquad A^5 = \begin{pmatrix} 0 & -\alpha^5/c^4 \\ \alpha^5/c^6 & 0 \end{pmatrix}$$

And the elements of matrix exponential curve take the form

$$\left[\exp(vA)\right]_{11} = \left[\exp(vA)\right]_{22} = I - \frac{v^2}{2!c^2} + \frac{v^4}{4!c^4} - ....,$$

$$\left[\exp(vA)\right]_{21} = -c^2\left[\exp(vA)\right]_{12} = \frac{\alpha}{c}\left(\frac{v}{c}I - \frac{v^3}{3!c^3} + \frac{v^5}{5!c^5} - .....\right),$$

where I is the unit matrix 2×2. It is not difficult to see that $-\alpha^2 = \alpha^4 = -\alpha^6 = \alpha^8 = ... = I$, therefore we finally obtain

$$\exp(vA) = \begin{pmatrix} Ich\ v/c & -c\alpha sh\ v/c \\ (\alpha sh\ v/c)/c & Ich\ v/c \end{pmatrix} =$$

$$\begin{pmatrix} ch\ v/c & 0 & 0 - csh\ v/c \\ 0 & ch\ v/c & csh\ v/c & 0 \\ 0 & (ch\ v/c)/c & ch\ v/c & 0 \\ -(sh\ v/c)/c & 0 & 0 & ch\ v/c \end{pmatrix}.$$

Now we return to (4.10) and substituting there $\exp(vA)$, we find

$$E'_y = E_y ch\ v/c - cB_z sh\ v/c, \qquad E'_z = E_z ch\ v/c + cB_y sh\ v/c,$$
$$B'_y = B_y ch\ v/c + (E_z/c)sh\ v/c, \ B'_z = B_z ch\ v/c - (E_y/c)sh\ v/c ,$$

or in the vector record

$$\vec{E}'_\perp = \vec{E}_\perp ch\ \frac{v}{c} + \frac{v}{c}\vec{v}\times\vec{B}_\perp sh\frac{v}{c},$$
$$\vec{B}'_\perp = \vec{B}_\perp ch\ \frac{v}{c} - \frac{1}{vc}\vec{v}\times\vec{E}_\perp sh\frac{v}{c}, \quad (4.11)$$

This is conversions (4.5).

If(4.4) is expanded in a series, we obtain

$$\varphi'(r, v_\perp) = \frac{ech\frac{v_\perp}{c}}{4\pi\varepsilon r} = \varphi(r)ch\frac{v_\perp}{c} = \varphi(r)\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right).$$

As can be seen, it differs from the ratio (2.17) is only factor $\frac{1}{2}$.

5. Conclusion

Coulomb law determines the static force, which acts between two fixed charges. However, this law does not give the answer, how this force will change, if charges will accomplish relative motion. Answer to this question can be obtained from the laws of induction. In the article it is shown that also the scalar potential and the electric field of charge at the observation point depend on relative motion of charge relative to this point.

References

[1] R. Feynman, R. Leighton, M. Sends, Feynman lectures on physics, –M..Mir, Vol. 6(1977).

[2] F. F. Mende, On refinement of equations of electromagnetic induction,– Kharkov, deposited in VINITI, No 774 – B88 Dep.,1988.

[3] F. F. Mende, Are there errors in modern physics. Kharkov, Constant,2003.

[4] F. F. Mende. On refinement of certain laws of classical electrodynamics, arXiv, physics/0402084.

[5] F. F. Mende. Conception of the scalar-vector potential in contemporary electrodynamics, arXiv.org/abs/physics/0506083.

[6] F. F. Mende, Great misconceptions and errors physicists XIX-XX centuries. Revolution in modern physics, Kharkov NTMT, 2010.

[7] F. F. Mende New electrodynamics. Revolution in the modern physics. Kharkov, NTMT, 2012.

[8] F. F. Mende, New approaches in contemporary classical electrodynamics. Part II, Engineering Physics, №2, 2013, p. 3-17.

[9] S.Ramo, John. Winner. Fields and Waves in modern electronics. OGIZ: 1948.

[10] F. F. Mende. The Classical Conversions of Electromagnetic Fields on Their Consequences. AASCIT Journal of Physics. Vol. 1, No. 1, 2015, pp. 11-18.